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Improvements to the Solution of the Viscous Shock Layer Equations

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An analysis and numerical solution of the fully viscous shock layer equations for hypersonic flow past blunt nosed bodies is presented. Solutions were obtained using a numerical scheme developed to solve the governing equations simultaneously as a coupled set of equations. Through application of the method to hypersonic flow past hyperboloids, it was found that the method is stable and produces results in good comparison with other methods. The main success of the method lies in its ability of extending the flow field calculations far aft of the nose region.

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PREFACE

The work reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by the Department of Aerospace Engineering and Applied Mechanics, University of Cincinnati, Cincinnati, Ohio 45221, under Contract F40600-77-C-0001. The Air Force project manager was Elton R. Thompson, DOTR.

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I. INTRODUCTION

The calculation of viscous flow fields past high speed axisymmetric blunt bodies is of prime interest to the designer of the reentry space vehicles. The variety of conditions met during a vehicle reentry requires the solution to be valid over a wide range of Reynolds number, ranging from a low Reynolds number at high altitude to a high Reynolds number at low altitude.

For high Reynolds number flows past blunt bodies, a number of effective methods have been developed for analyzing the inviscid flow and the associated boundary layer flow over the body. In flow regimes of intermediate to low Reynolds numbers, the viscous region incompasses a significant portion of the shock layer and the classical boundary-layer approaches can not be used. The idea of using the second order boundary layer equations [1] to compute the flow field becomes appealing in such cases; however, this approach leads to several computational difficulties. Aside from the excessive computing time, one experiences problems with the strong vorticity interaction region which causes difficulty in the matching procedure between the viscid and inviscid region.

Because of the difficulties mentioned above, it is desirable to seek an alternate method of approach to the problem. The use of the full Navier-Stokes equations [2, 3] has been successful in providing solutions for the stagnation regions but generally have been applied for only one to two nose radius downstream. The complexity of the solution procedures of the Navier-Stokes

equations restrict their applications especially in the downstream direction. Because of this difficulty, attention has lately been turned toward the viscous shock layer techniques. The governing equations of these techniques are basically parabolic in nature and their numerical solution is a straightforward streamwise marching procedure.

The viscous shock layer equations, which were developed primarily by Davis [4], contain all of the terms in the Navier Stokes equations which contribute to second order effects in both the inviscid and viscid regions. If the pressure gradient normal to the body surface is assumed to be established entirely by centrifugal effects, the thin shock layer version of the equations is obtained. While the solution of the thin layer equation is straightforward with techniques similar to the boundary layer marching techniques, the solution of the full shock layer has encountered difficulties. In an attempt to solve the full layer, Davis developed a method wherein the solution is obtained through a relaxation process from the thin shock layer solution. While this method was successful, it is sensitive and can encounter divergent behavior in the relaxation scheme used to relieve the thin shock layer assumptions [5]. This difficulty is most severe for slender bodies. In an attempt to overcome this problem, a different relaxation technique was developed by Werle et al. [5]. This technique utilized an artificial time coordinate to relax the shock from an initial shape. Limited success has been reported when under-relaxation factors were used to remove instabilities that occurred in the

solution far downstream from the stagnation point.

The Werle et al. [5] and Davis [4] methods are both essentially relaxing the shock shape to take into account downstream influences and consequently acknowledge the boundary value nature of the problem. In both cases, difficulties were encountered whenever the shock layer thickness is large and especially far downstream on blunt slender bodies where the inviscid region encompasses a significant portion of the total shock layer thickness. These difficulties do not stem from the shock shape relaxing technique but rather occur due to other fundamental reasons.

It is known from small disturbance theory in hypersonic flow [6] that the streamwise velocity variation does not affect the solution of the other flow variables since the group of equations representing the continuity, normal momentum and energy are completely uncoupled from the longitudinal momentum. Therefore a similar behavior of the shock layer equations is expected in the inviscid region far downstream on a slender blunt body.

Far downstream on a slender body, the boundary layer is comparatively thin and the flow in the shock layer is predominately inviscid. A general description of the inviscid flow could be obtained from the solution of the inviscid shock layer equations. For slender bodies with relatively small surface slope, the approximations of hypersonic small disturbance theory is quite important. If these approximations are applied to the inviscid shock layer equations, and only first

order terms are kept, the normal momentum, continuity and energy equations become uncoupled from the longitudinal momentum equation. It is therefore clear that in this case the solution of the normal momentum, continuity and energy equations will not depend strongly on the solution of the tangential momentum equation.

All numerical methods for handling the shock layer equations solve the governing equations in a successive manner where the solution of the tangential momentum equation drives the solution of the normal momentum and continuity equations. These numerical methods become improper for the flow far downstream, since the tangential momentum equation does not have a predominant influence on the solution. A more adequate solution technique is to solve the equations simultaneously; this will ensure the proper coupling between each equation in the development of the solution.

In reference [5], difficulties in the solution were reported as occurring in the iterative loop that is used to solve the continuity and the normal momentum equations. This difficulty was reported as an oscillatory behavior of the solution from one iteration to next. It was suggested in the same reference that the continuity and normal momentum equations should be solved simultaneously as a coupled set to overcome this misbehavior. Waskiewicz et al. [7] have applied this suggestion and have reported good improvement in the solution obtained on a sphere. It seems that the coupling has strengthened the mutual

dependence of the continuity and normal momentum equations.

In general, better solutions are expected if all the governing equations are solved simultaneously as a coupled set, as the interaction and dependence of the equations on each other will be automatically guaranteed and the physical nature of the problem will be automatically satisfied. While solving the equations as a coupled set has shown excellent results in handling inviscid shock layer equations [8, 9], it has not yet been used in the viscous shock layer equations. The aim of the present study is to develop a numerical algorithm that is capable of solving two first order and two second order equations simultaneously and to assess its applicability to the solution of the full viscous shock layer equations.

II. GOVERNING EQUATIONS

The viscous shock layer equations used in this study have been developed by Davis [4]. A detailed derivation of these equations are given in references [4] and [10], and they are only summarized here. The Navier Stokes equations are first written in a boundary layer coordinate system (see Fig. 1). Terms higher than second order, in the inverse square root of the Reynolds number, from both a viscid and an inviscid viewpoint are then neglected. The resulting equations are:

Continuity:

$$[(r+n\cos\phi)^j \rho u]_s + [(1+\kappa n)(r+n\cos\phi)^j \rho v]_n = 0 \tag{1a}$$

Longitudinal Momentum:

$$\rho\{u\, u_s/(1+\kappa n) + v\, u_n + \kappa uv/(1+\kappa n)\} + p_s/(1+\kappa n)$$
$$= [\varepsilon^2/(1+\kappa n)^2 (r+n\cos\phi)^j][(1+\kappa n)^2 (r+n\cos\phi)^j \tau]_n \tag{1b}$$

where,

$$\tau = \mu[u_n - \kappa u/(1+\kappa n)] \tag{1c}$$

Normal Momentum:

$$\rho\{u\, v_s/(1+\kappa n) + v\, v_n - \kappa u^2/(1+ n)\} + p_n = 0 \tag{1d}$$

which with the thin shock layer approximation becomes,

$$-\rho\, \kappa u^2/(1+\kappa n) + p_n = 0 \tag{1e}$$

Energy Equation:

$$\rho\{uT_s/(1+\kappa n) + vT_n\} - u\, p_s/(1+\kappa n) - v\, p_n = \varepsilon^2\tau^2/\mu$$

$$+ [\varepsilon^2/(1+\kappa n)(r+n\cos\phi)^j][(1+\kappa n)(r+n\cos\phi)^j q]_n \qquad (1f)$$

where

$$q = \mu\, T_n/\sigma \qquad (1g)$$

Equation of State:

$$p = (\gamma-1)\rho T/\gamma \qquad (1h)$$

Viscosity Law:

$$\mu = T^{3/2}\,(1+c')/(T+c') \qquad (1i)$$

where

$$c' = c^*/M_\infty^2\, T_\infty^*\,(\gamma-1)$$

and c^* is taken to be 198.6°R for air.

The surface boundary conditions are the no slip conditions:

$$u(s,o) = v(s,o) = 0 \qquad (2a)$$

and

$$T(s,o) = T_w \qquad (2b)$$

while at the shock location the oblique shock relations are used. These relations are given as:

$$u_{sh} = \tilde{u}_{sh}\sin(\alpha+\beta) + \tilde{v}_{sh}\cos(\alpha+\beta) \qquad (3a)$$

$$v_{sh} = -\tilde{u}_{sh}\cos(\alpha+\beta) + \tilde{v}_{sh}\sin(\alpha+\beta) \qquad (3b)$$

where $\tilde{u}_{sh}$ and $\tilde{v}_{sh}$ are velocity components at the shock given as

$$\tilde{u}_{sh} = \cos\alpha \tag{3c}$$

$$\tilde{v}_{sh} = - \sin\alpha/\rho_{sh} \tag{3d}$$

and

$$\rho_{sh} = \gamma p_{sh}/(\gamma-1)T_{sh} \tag{3e}$$

$$p_{sh} = [2/(\gamma+1)]\sin^2\alpha - (\gamma-1)/\gamma(\gamma+1)M_\infty^2 \tag{3f}$$

$$\begin{aligned} T_{sh} = {} & (\tilde{u}_{sh} - \cos\alpha)^2/2 + \{[4\gamma/(\gamma+1)^2]\sin^2\alpha + [2/(\gamma-1) \\ & - 4(\gamma-1)/(\gamma+1)^2]/M_\infty^2 - 4/(\gamma+1)^2 \ M_\infty^4 \ \sin^2\alpha\}/2 \end{aligned} \tag{3g}$$

For ease in the numerical computations and also for shock relaxing considerations, the above equations are normalized by using the following variables:

$$\begin{aligned} &\eta = n/n_{sh} \quad && \xi = s \quad && \bar{u} = u/u_{sh} \quad && \bar{v} = v/v_{sh} \\ &\bar{t} = T/T_{sh} \quad && \bar{p} = p/p_{sh} \quad && \bar{\rho} = \rho/\rho_{sh} \quad && \bar{\mu} = \mu/\mu_{sh} \end{aligned} \tag{4a-h}$$

The resulting equations are two second order and two first order differential equations and are given by:

s-Momentum Equation:

$$\partial^2\bar{u}/\partial\eta^2 + \alpha_1(\partial\bar{u}/\partial\eta) + \alpha_2\bar{u} + \alpha_3 + \alpha_4(\partial\bar{u}/\partial\xi) = 0 \tag{5}$$

where

$$\alpha_1 = \frac{\rho_{sh} u_{sh} n'_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{\overline{\rho}\overline{u}\eta}{\overline{\mu}} - \frac{\rho_{sh} v_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{\overline{\rho}\overline{v}}{\overline{\mu}}$$

$$+ \overline{\mu}_\eta/\overline{\mu} + \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi n_{sh}}{\gamma+n_{sh}\eta\cos\phi} \tag{5a}$$

$$\alpha_2 = - \frac{\rho_{sh} u'_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{\overline{\rho}\overline{u}}{\overline{\mu}} - \frac{\rho_{sh} v'_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} \overline{\rho} \frac{\overline{v}}{\overline{\mu}}$$

$$- \kappa \frac{n_{sh}}{1+\kappa n_{sh}\eta} \overline{\mu}_\eta/\overline{\mu} - \left(\frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi n_{sh}}{r+n_{sh}\eta\cos\phi}\right) \times \left(\frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta}\right) \tag{5b}$$

$$\alpha_3 = - \frac{p_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}} 1/\overline{\mu}\ 1/u_{sh} \left(\overline{p}_\xi - \frac{n'_{sh}}{n_{sh}} \eta \overline{p}_\eta + \frac{p'_{sh}}{p_{sh}} \overline{p}\right) \tag{5c}$$

$$\alpha_4 = - (\rho_{sh} u_{sh} n_{sh}/\varepsilon^2 \mu_{sh})(n_{sh}/(1+\kappa n_{sh}\eta) \frac{\overline{\rho}\overline{u}}{\overline{\mu}} \tag{5d}$$

Energy Equation:

$$\partial^2\overline{t}/\partial\eta^2 + \tilde{\alpha}_1 (\partial\overline{t}/\partial\eta) + \tilde{\alpha}_2 \overline{t} + \tilde{\alpha}_3 + \tilde{\alpha}_4(\partial\overline{t}/\partial\xi) = 0 \tag{6}$$

where

$$\tilde{\alpha}_1 = \frac{\rho_{sh} u_{sh} n'_{sh} \sigma}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{\overline{\rho}\overline{u}\eta}{\overline{\mu}} - \frac{\rho_{sh} v_{sh} n_{sh} \sigma}{\varepsilon^2 \mu_{sh}} \times \frac{\overline{\rho}\overline{v}}{\overline{\mu}}$$

$$+ \overline{\mu}_\eta/\overline{\mu} + \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi n_{sh}}{r+n_{sh}\eta\cos\phi} \tag{6a}$$

$$\tilde{\alpha}_2 = -(\rho_{sh} u_{sh} T'_{sh}/\varepsilon^2 \mu_{sh} T_{sh})(n^2_{sh}/(1+\kappa n_{sh}\eta)) \times \frac{\overline{\rho}\overline{u}}{\overline{\mu}} \tag{6b}$$

$$\tilde{\alpha}_3 = \frac{\rho_{sh} u_{sh} n_{sh} \sigma}{\epsilon^2 \mu_{sh} T_{sh}} \, 1/\bar{\mu} \, [\frac{n_{sh} \bar{u}}{1+\kappa n_{sh} \eta} \, (\bar{p}_\xi - \frac{n'_{sh}}{n_{sh}} \eta \bar{p}_\eta + \frac{p'_{sh}}{p_{sh}} \bar{p})$$

$$+ \frac{v_{sh}}{u_{sh}} \bar{v} \bar{p}_\eta] + \frac{u_{sh}^2 \sigma}{T_{sh}} \, (\bar{u}_\eta - \frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} \bar{u})^2 \tag{6c}$$

$$\tilde{\alpha}_4 = - (\sigma \rho_{sh} u_{sh} n_{sh} / \epsilon^2 \mu_{sh}) (n_{sh} / (1+\kappa n_{sh} \eta)) \, \frac{\bar{\rho} \bar{u}}{\bar{\mu}} \tag{6d}$$

Continuity Equation:

$$[n_{sh} (r + n_{sh} \eta \cos\phi) \, \rho_{sh} u_{sh} \, \bar{\rho} \bar{u}]_\xi + [(r + n_{sh} \eta \cos\phi) x$$

$$\{(1+\kappa n_{sh} \eta) \rho_{sh} v_{sh} \, \bar{\rho} \bar{v} - n'_{sh} \, \rho_{sh} u_{sh} \, \bar{\rho} \bar{u} \eta\}]_\eta = 0 \tag{7}$$

n-Momentum Equation:

$$\frac{\bar{\rho} \bar{u}}{(1+\kappa n_{sh} \eta)} \, (\bar{v}_\xi - n'_{sh} / n_{sh} \, \eta \bar{v}_\eta + \frac{v'_{sh}}{v_{sh}} \bar{v}) + \frac{v_{sh}}{u_{sh}} \frac{\bar{\rho} \bar{v}}{n_{sh}} \bar{v}_\eta$$

$$- \frac{\kappa}{1+\kappa n_{sh}} \, \frac{u_{sh}}{v_{sh}} \, \bar{\rho} \bar{u}^2 + \frac{p_{sh}}{\rho_{sh} u_{sh} v_{sh} n_{sh}} \bar{p}_\eta = 0 \tag{8}$$

where with the thin layer approximation this equation becomes,

$$\bar{p}_\eta = [\kappa / (1+\kappa n_{sh} \eta)] (\rho_{sh} u_{sh}^2 n_{sh} / p_{sh}) \, \bar{\rho} \bar{u}^2 \tag{8a}$$

The remaining equations are:

The Equation of State:

$$\bar{p} = \bar{\rho} \bar{t} \tag{9}$$

and the viscosity law,

$$\bar{\mu} = [(T_{sh} + c')/(T_{sh}\bar{t} + c')]\ \bar{t}^{3/2} \tag{10}$$

The surface boundary conditions are

$$\bar{u} = 0, \quad \bar{v} = 0, \quad \text{and} \quad \bar{t} = \bar{t}_w \tag{11}$$

and the conditions at the shock are:

$$\bar{u} = \bar{v} = \bar{t} = \bar{p} = \bar{\rho} = \bar{\mu} = 1 \tag{12}$$

An equation of global mass conservation can be obtained from equation (7) by integrating from $\eta = 0$ to $\eta = 1$ while holding ξ constant. This results in

$$\frac{dm}{d\xi} = (r+n_{sh}\cos\phi)\ [n'_{sh}\rho_{sh}u_{sh} - (1+\kappa n_{sh})\rho_{sh}v_{sh}] \tag{13a}$$

where

$$m = \int_0^1 n_{sh}(r+n_{sh}\eta\cos\phi)\ \rho_{sh}u_{sh}\ \bar{\rho}\bar{u}\ d\eta \tag{13b}$$

is proportional to the rate of mass flux between the body and shock at a given position on the body surface. A limiting form of equations (13a) and (13b) is obtained at the stagnation point by applying the condition $\xi = 0$ and using L'Hopitals Rule. This limiting form is used to determine the shock stand-off distance at the stagnation point.

Equations (5) to (10) constitute the complete set of governing equations for the unknowns $\bar{u}$, $\bar{v}$, $\bar{t}$, $\bar{p}$, $\bar{\mu}$ and $\bar{\rho}$. These equations are solved along with the surface conditions given by equation (11) and the shock conditions given by equation (12).

III. THE "UNSTEADY" SHOCK LAYER EQUATIONS

Werle et al. [5] have developed a time relaxation scheme where an initial guess of the shock shape is relaxed in an artificial time like manner toward the "steady state" solution. The scheme was developed to overcome the divergent behavior in the iteration scheme developed by Davis [4]. Two versions, but essentially the same, have been developed in which the first relaxes the shock wave thickness [5], while the second relaxes the shock radius [11]. The first approach is very suitable and directly handles the variable which appears always in the equations. But the second is more general and it could handle cases that involve discontinuity in the surface curvature. For reasons of generality, the second version will be adopted and will be summarized here.

In order to demonstrate the adopted time relaxation technique, the "unsteady" shock layer equations should first be obtained. In reference [5], Werle et al. have reasoned that an artificial time term can be added to the equations through the shock variable derivatives. For this reason, the governing equations have to be in a normalized form to provide explicitly the shock derivatives. In Appendix A, the shock derivatives are given in terms of the shock slope ($d\alpha/ds$). Since $d\alpha/ds$ represents the curvature of the shock shape, a relation between $d\alpha/ds$ and the second derivative of the shock radius is derived in Appendix B and is given by

$$\frac{d\alpha}{ds} = \frac{d^2R}{ds^2}\left[\frac{\cos^2(\alpha-\phi)}{(1+\kappa n_{sh})\cos\phi}\right] - \frac{dR}{ds}\left[\frac{\kappa\sin(2\alpha-2\sigma)}{\cos\phi(1+\kappa n_{sh})} + \frac{n_{sh}\kappa'\cos^2(\alpha-\phi)}{\cos\phi(1+\kappa n_{sh})^2}\right] \tag{14}$$

This relation differs from that of reference [11] in including the effect of the variation of the body curvature (κ'). The artificial time like term is then introduced to give

$$\frac{d\alpha}{ds} = \left[\frac{\partial^2 R}{\partial s^2} - \frac{\partial R}{\partial t}\right]\left[\frac{\cos^2(\alpha-\phi)}{(1+\kappa n_s)\cos\phi} - \frac{dR}{ds}\left[\frac{\kappa\sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_s)} + \frac{n_{sh}\kappa'\cos^2(\alpha-\phi)}{\cos\phi(1+\kappa n_{sh})^2}\right]\right. \tag{15}$$

At the stagnation point $\phi = 90°$ and this equation becomes singular. However, if the limit is taken of this equation as s tends to zero, the following equation is obtained

$$\frac{d\alpha}{ds} = \frac{1}{1+n_{sh}}\left[\frac{\partial^2 n_{sh}}{\partial s^2} - \frac{1}{3}\frac{\partial n_{sh}}{\partial t}\right] - 1 \tag{16}$$

Upon substitution for $d\alpha/ds$ in the shock derivative relations given in Appendix A and using them in the governing equations, the "unsteady" equations are achieved.

In summary, equations (5) to (8) together with equations (A1, A2, A3, A4) and equation (15) represents the set of "unsteady" shock layer equations. The set is a form suitable for numerical computations. However, for time relaxation considerations, it is necessary to write the longitudinal momentum in its final form given by

$$\frac{\partial^2\bar{u}}{\partial\eta^2} + \beta_1\frac{\partial\bar{u}}{\partial\eta} + \beta_2\left[\frac{\partial^2 R}{\partial s^2} - \frac{\partial R}{\partial t}\right] + \beta_3\frac{\partial R}{\partial s} + \beta_4 + \beta_5\frac{\partial\bar{u}}{\partial\xi} = 0 \tag{17}$$

where β_1, β_2, β_3, β_4 and β_5 are obtained from reference [11] and are listed here in Appendix C.

IV. NUMERICAL ANALYSIS

1. General Considerations:

The "unsteady" governing equations are solved using a time relaxing approach equivalent to the implicit alternating direction method. This approach utilizes two steps in which the first yields the flow properties in the shock layer while the second step is to update the shock shape itslef. This is demonstrated by writing the "unsteady" longitudinal momentum equation (17) in two time steps:

First Step (Star Sweep): from t^n to $t^* = t^n + \frac{\Delta t}{2}$

$$\frac{\partial^2 \bar{u}^*}{\partial \eta^2} + \beta_1^* \frac{\partial \bar{u}^*}{\partial \eta} + \beta_2^* \left[\frac{\partial^2 R^n}{\partial s^2} - \frac{\partial R^*}{\partial t}\right] + \beta_3^* \frac{\partial R^n}{\partial s} + \beta_4^* + \beta_5^* \frac{\partial \bar{u}^*}{\partial \xi} = 0 \quad (18)$$

Second Step (Final Sweep): from t^* to $t^{n+1} = t^* + \frac{\Delta t}{2}$

$$\beta_2^* \frac{\partial^2 R^{n+1}}{\partial s^2} - \beta_2^* \frac{\partial R^{n+1}}{\partial t} + \beta_3^* \frac{\partial R^{n+1}}{\partial s} + \left[\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} + \beta_5 \frac{\partial \bar{u}}{\partial \xi} + \beta_4\right]^* = 0 \quad (19)$$

Using equation (C6) of Appendix C, equation (19) can be written in a form independent of η by using equation (18) to give

$$\frac{\partial^2 R^{n+1}}{\partial s^2} - \left[2\kappa \tan(\alpha-\phi) + \frac{n_{sh}\kappa'}{1+\kappa n_{sh}}\right] \frac{\partial R^{n+1}}{\partial s} - \frac{2}{\Delta t} R^{n+1}$$

$$- \frac{\partial^2 R^n}{\partial s^2} + \left[2\kappa \tan(\alpha-\phi) + \frac{n_{sh}\kappa'}{1+\kappa n_{sh}}\right] \frac{\partial R^n}{\partial s}$$

$$+ \frac{2}{\Delta t} (2R^* - R^n) \quad (20)$$

The boundary conditions associated with the star sweep of the unsteady equations are typical no slip conditions (11) at the surface and Rankine-Hugoniot conditions at the shock location (12). The boundary conditions associated with the final sweep are the same as those used in reference [11] and are given as:

$$\text{at} \quad s = 0 \qquad R^{n+1} = 0 \tag{21a}$$

$$\text{at} \quad s = s_{max} \qquad R^{n+1}_{max} = R^{*}_{max} \tag{21b}$$

2. <u>Method of Solution of the Star Sweep</u>:

The deviation point between the present work and that of references [4] and [5] is the way the equations are solved during the star sweep. Previously, the equations were solved in a successive manner where the solution of one equation drives the solution of the other. This technique is widely used in many numerical calculations and is known as the cascading scheme. In the present work, the equations will be solved simultaneously. This technique will be referred as the coupling scheme. In reference [7] a combination of the two schemes were used where the two second order equations, s-momentum and energy, are solved in a successive manner while the continuity and n-momentum are solved simultaneously. This combination has indicated an improvement in the solution as discussed earlier in the report.

The equations to be solved, during the star sweep, are the s-momentum, energy, n-momentum and the continuity. When the equation of state is used to replace the density variable by the pressure and temperature variables, the governing equations are

mainly two first order and two second order equations in the unknowns $\bar{u}$, $\bar{v}$, $\bar{p}$ and $\bar{t}$. They require six boundary conditions, three on the body surface and three at the shock. Actually, there are seven boundary conditions available, four at the shock and three at the body surface, but there is an additional unknown n_{sh}. Thus the equations and the boundary conditions form a complete system of equations. In this solution procedure, n^*_{sh} is assumed to be known and is iterated on to satisfy the extra boundary condition. In this case, the extra boundary condition is chosen to be $\bar{v}(\eta=1) = 1$.

The governing equations (5)-(8) are nonlinear equations. It is first convenient to linearize the governing equations with the following relations, where the quantities with superscript ° are evaluated from previous iteration

$$XY = X^\circ Y + Y^\circ X - X^\circ Y^\circ \qquad (22a)$$

$$XYZ = X^\circ Y^\circ Z + Y^\circ Z^\circ X + Z^\circ X^\circ Y - 2\, X^\circ Y^\circ X^\circ \qquad (22b)$$

$$\frac{XZ}{Y} = \frac{X^\circ}{Y^\circ} Z + \frac{Z^\circ}{Y^\circ} X - \frac{X^\circ Z^\circ}{Y^{\circ 2}} Y \qquad (22c)$$

Where X, Y and Z can represent any of the variables $\bar{u}$, $\bar{v}$, $\bar{p}$ and $\bar{t}$ and their derivatives. If X°, Y°, Z° values are taken from previous ξ solution, the linearization relations are second order in $\Delta\xi$ and nonlinearity iterations are not necessary (This method is known as quasi-linearization).

The resulting linear equations, after dropping the superscript $^-$, are:

s-Momentum:

$$\frac{\partial^2 u}{\partial \eta^2} + (a_1 u_\xi + a_3 p_\xi) + (b_1 u_\eta + b_3 p_\eta)$$

$$+ (c_1 u + c_2 v + c_3 p + c_4 t) + d = 0 \qquad (23)$$

Energy:

$$\frac{\partial^2 t}{\partial \eta^2} + (a_3 p_\xi + a_4 t_\xi) + (b_1 u_\eta + b_3 p_\eta + b_4 t_\eta)$$

$$+ (c_1 u + c_2 v + c_3 p + c_4 t) + d = 0 \tag{24}$$

η-Momentum:

$$a_2 v_\xi + (b_2 v_\eta + b_3 p_\eta) + (c_1 u + c_2 v + c_3 p + c_4 t) + d = 0 \tag{25}$$

Continuity:

$$(a_1 u_\xi + a_3 p_\xi + a_4 t_\xi) + (b_1 u_\eta + b_2 v_\eta + b_3 p_\eta + b_4 t_\eta)$$

$$+ (c_1 u + c_2 v + c_3 p + c_4 t) + d = 0 \tag{26}$$

The coefficients a_1, a_2, ... are evaluated independently for each individual equation and are given in Appendix D. To solve these equations numerically, it is necessary to write these equations in a finite difference form. Let the subscript m denote the station measured along the body, while the subscript n denotes the station measured normal to the body, the derivatives in the η direction are:

$$\left(\frac{\partial w}{\partial \eta}\right)_{m,n} = [\Delta\eta_{n-1}/\Delta\eta_n (\Delta\eta_n + \Delta\eta_{n-1})] w_{m,n+1}$$

$$+ [(\Delta\eta_n - \Delta\eta_{n-1})/\Delta\eta_n \Delta\eta_{n-1}] w_{m,n} - [\Delta\eta_n/\Delta\eta_{n-1}(\Delta\eta_n + \Delta\eta_{n-1})]$$

$$w_{m,n-1} - \frac{1}{6}\left(\frac{\partial^3 w}{\partial \eta^3}\right)_{m,n} \Delta\eta_n \Delta\eta_{n-1} \tag{27}$$

$$\left(\frac{\partial^2 w}{\partial \eta^2}\right)_{m,n} = [2/\Delta\eta_n(\Delta\eta_n + \Delta\eta_{n-1})]\ w_{m,n+1} - (2/\Delta\eta_n\Delta\eta_{n-1})\ w_{m,n}$$

$$+ [2/\Delta\eta_{n-1}(\Delta\eta_n+\Delta\eta_{n-1})w_{m,n-1} - \frac{1}{12}\left(\frac{\partial^4 w}{\partial \eta^4}\right)_{m,n} \Delta\eta_n\Delta\eta_{n-1}$$

$$- \frac{1}{3}\left(\frac{\partial^3 w}{\partial \eta^3}\right)_{m,n} (\Delta\eta_n-\Delta\eta_{n-1}) - \frac{1}{12}\left(\frac{\partial^4 w}{\partial \eta^4}\right)_{m,n} (\Delta\eta_n-\Delta\eta_{n-1})^2 \quad (28)$$

While the derivative in ξ direction is a two point difference given by:

$$\frac{\partial w}{\partial \xi} = (w_{m,n} - w_{m-1,n})/\Delta\xi \quad (29)$$

The η derivatives are of second order accurate in the step size if constant $\Delta\eta$ is chosen. If the ξ derivative is evaluated at mid point $(m-\frac{1}{2},n)$ and other terms are averaged, one obtains a Crank-Nicolson scheme. If a backward difference is used for $\partial w/\partial\xi$ at the (m,n) a purely implicit scheme is generated.

The previous difference quotients are suitable for the second order s-momentum and energy equations. For the first order n-momentum and continuity equations, it is convenient to use two point difference relations for both η and ξ derivatives as follows:

$$\left(\frac{\partial w}{\partial \eta}\right)_{m,n-\frac{1}{2}} = (w_{m,n} - w_{m,n-1})/\Delta\eta_{n-1} + 0(\Delta\eta^2_{n-1}) \quad (30)$$

$$\frac{\partial w}{\partial \xi} = (w_{m,n-\frac{1}{2}} - w_{m-1,n-\frac{1}{2}})/\Delta\xi \quad (31)$$

If the ξ derivative is evaluated at $(m-\frac{1}{2},n-\frac{1}{2})$ and $\frac{\partial w}{\partial \eta}$ is averaged between the stations m-1 and m, the box scheme is obtained.

If the linearized forms of s-momentum (23), energy (24), n-momentum (25) and continuity (26) are evaluated at $(m-\frac{1}{2},n)$, $(m-\frac{1}{2},n)$, $(m-\frac{1}{2},n-\frac{1}{2})$ and $(m-\frac{1}{2},n+\frac{1}{2})$ respectively, the resulting difference equations will be in a form suitable for solution with an algorithm similar to Thomas algorithm. Using the difference quotients (27)-(29) and (30)-(31), these difference equations are:

$$(a_{11}u_{n-1} + a_{13}p_{n-1}) + (b_{11}u_n + b_{12}v_n + b_{13}p_n + b_{14}t_n) + (c_{11}u_{n+1} + c_{13}p_{n+1}) = d_1 \tag{32}$$

$$(a_{21}u_{n-1} + a_{23}p_{n-1} + a_{24}t_{n-1}) + (b_{21}u_n + b_{22}v_n + b_{23}p_n + b_{24}t_n) + (c_{21}u_{n+1} + c_{23}p_{n+1} + c_{24}t_{n+1}) = d_2 \tag{33}$$

$$(b_{31}u_n + b_{32}v_n + b_{33}p_n + b_{34}t_n) + (c_{31}u_{n+1} + c_{32}v_{n+1} + c_{33}p_{n+1} + c_{34}t_{n+1}) = d_3 \tag{34}$$

and

$$(a_{41}u_{n-1} + a_{42}v_{n-1} + a_{43}p_{n-1} + a_{44}t_{n-1}) + (b_{41}u_n + b_{42}v_n + b_{43}p_n + b_{44}t_n) = d_4 \tag{35}$$

where $n = 2, 3, \ldots. (N-1)$.

The coefficients in the above equations vary from one grid point to another and are given in Appendix E. These linear

algebraic equations are to be solved with the following boundary conditions,

$$u_1 = 0 \tag{36a}$$

$$v_1 = 0 \tag{36b}$$

$$t_1 = t_w \tag{36c}$$

$$u_N = 1 \tag{37a}$$

$$v_N = 1 \tag{37b}$$

$$p_N = 1 \tag{37c}$$

$$t_N = 1 \tag{37d}$$

The difference equations together with the boundary conditions lend themselves to solution by the following recursion relations:

$$u_{n+1} = D_{u_{n+1}} u_n + E_{u_{n+1}} v_n + F_{u_{n+1}} p_n + G_{u_{n+1}} t_n + H_{u_{n+1}} \tag{38a}$$

$$v_{n+1} = D_{v_{n+1}} u_n + E_{v_{n+1}} v_n + F_{v_{n+1}} p_n + G_{v_{n+1}} t_n + H_{v_{n+1}} \tag{38b}$$

$$p_{n+1} = D_{p_{n+1}} u_n + E_{p_{n+1}} v_n + F_{p_{n+1}} p_n + G_{p_{n+1}} t_n + H_{p_{n+1}} \tag{38c}$$

$$t_{n+1} = D_{t_{n+1}} u_n + E_{t_{n+1}} v_n + F_{t_{n+1}} p_n + G_{t_{n+1}} t_n + H_{t_{n+1}} \tag{38d}$$

The two subscripts marking the coefficients D, E, F, G and H denote to which variable and grid point they belong. The solution steps using these recursion formulas are given in Appendix F.

3. <u>Solution of the Final Sweep Equation</u>:

The governing equation of this step, at a fixed time, is of the form

$$\frac{d^2R^{n+1}}{dS^2} + \alpha_1 \frac{dR^{n+1}}{dS} + \alpha_2 R^{n+1} + \alpha_3 = 0 \tag{39}$$

where
$$\alpha_1 = -[2\kappa \tan(\alpha-\phi) + \frac{n_{sh}\kappa'}{1+\kappa n_{sh}}]$$

$$\alpha_2 = - 2/\Delta t$$

$$\alpha_3 = - \frac{\partial^2 R^n}{\partial S^2} + [2\kappa \tan(\alpha-\phi) + \frac{n_{sh}\kappa'}{1+\kappa n_{sh}}] \frac{\partial R^n}{\partial s} + \frac{2}{\Delta t} (2R^* - R^n)$$

with the boundary conditions

$$R^{n+1}(0) = 0$$

$$R^{n+1}(S_{max}) = R^*(S_{max}) \quad .$$

The problem is a two point boundary value problem and its numerical solution is straightforward since it reduces to a tridiagonal form once the following second order difference quotients are substituted into the differential equation (39):

$$\frac{d^2R}{dS^2} = (R_{m+1} - 2R_m + R_{m-1})/ \Delta S^2 \tag{40a}$$

$$\frac{dR}{dS} = (R_{m+1} - R_{m-1})/2\Delta S \tag{40b}$$

4. Overall Method of Solution:

The overall method of solution employed is as follows. An initial guess is first made for the shock shape over the region of interest. This guess can be made arbitrary and the most simple one is to assume the shock lies parallel to the body surface with a constant shock thickness n_{sh}^{n}. Based on this guess, the first and second derivatives of the shock radius are evaluated. The star sweep equations are then solved by starting at the stagnation point where the governing equations are reduced to ordinary differential equations. With initial guesses for the flow profiles and for n_{sh}^{*}, all the linearized governing equations are solved simultaneously with the numerical algorithm presented earlier to obtain the new flow profiles and n_{sh}^{*}. n_{sh}^{*} at each iteration is calculated from the equation representing the global mass conservation. The coefficients of the linearized equations are then reevaluated. Repetition of the above steps is continued until a converged solution is obtained at the stagnation point. The method then steps along the body surface. The previous station values of the flow profiles are used at each new step to evaluate the coefficients of the quasi-linearized equations. Their solution is then obtained by the same numerical algorithm. Iterations on the nonlinear terms is not necessary since the solution will be at least of second order accurate in $\Delta\xi$. However, local iterations at each ξ station are still necessary to obtain the proper n_{sh}^{*} which would satisfy the extra boundary condition $\bar{v}(\eta=1) = 1$ (or $v(\eta=1) = v_{sh}$).

A Newton-Raphson iteration procedure is utilized to obtain the shock distance n_{sh}^{*}.

Once the above procedure has marched over the entire mesh the final sweep is then solved for n_{sh}^{n+1}. No iteration of the final sweep equation is required since the equation is linear. The shock shape solution obtained is then used to initialize the following star sweep in time. This process is continued until two alternate final sweeps converged to a desired degree of accuracy. The solution obtained is the required "steady state" solution.

V. RESULTS AND DISCUSSION

The numerical algorithm developed in Section IV was applied to obtain the solution of the full viscous shock layer equations over blunt body configurations. The success of the algorithm lies in its ability to extend the flow field calculations far aft of the nose region without the oscillatory behavior reported in references [5] and [11]. Numerical solutions were successfully generated up to twenty times the nose radius downstream over hyperbolic blunt bodies. The algorithm was also capable of handling flow fields around slender bodies and this was demonstrated by obtaining solutions over hyperboloid bodies with small asymptotic angles. The reliability of the method was checked through comparisons with other numerical calculations and with available experimental data.

In all cases. the gas was assumed to be a perfect gas with a constant specific heat, constant Prandtl number and the viscosity was assumed to be given by the Sutherland law. The first case considered was that of flow over a 22.5° half angle hyperboloid. The flow conditions were the same as those considered in reference [5] with a free stream Mach number of $M_\infty = 21.75$, free stream temperature of $T_\infty = 351.8°R$, wall to stagnation temperature ratio of 0.05 and Reynolds number, based on the nose radius, of $Re_\infty = 430$.

Initially the shock was assumed parallel to the body surface and at a distance of 0.1072. This value is the converged stagnation point shock stand-off distance obtained using Davis' method.

The initial shock shape is in general not critical to the overall solution and other smooth initial shock shapes could be used as well. With the initial shock shape defined, local iterations were necessary to solve the star sweep equations for the new shock shape. Figure (2) shows the normal velocity component, $\tilde{v}$, at $\eta = 0.99$ at several s-locations for each iteration, where it is observed that the solution converges at all locations in less than six iterations. The solution did not only converge faster than the solution in reference [5], but also was more stable since the oscillatory behavior observed in the same reference was completely eliminated.

The choice of the time step size, in the alternating direction method, is analytically a hard task. In such cases numerical studies can be conducted to find the basis for the choice of the time step size. Figure (3) presents the results of a numerical study conducted to show the convergence of the shock radius with the global time iteration cycles for different time step sizes ranging from $\Delta t = 10$ to $\Delta t = 60$. If the solution is assumed converged when the change of the shock radius does not exceed 0.1% of its value, the figure indicates that this convergence limit is achieved faster with the largest time step size, $\Delta t = 60$. On the other hand, if the convergence limit is set to be 0.07%, the results show that this limit is achieved faster with the smallest time step size, $\Delta t = 10$. Therefore, the choice of the time step size will generally depend on the degree of accuracy desired.

In addition to the factor mentioned above, the consistency limitation of the alternating direction implicit method provides another factor in the time step size choice. The consistency limitation, as described in reference [5], shows that the two half time sweeps converge to slightly different values. Figures (4) and (5) represent the converged shock radius in the two sweeps for two different time step size values, namely $\Delta t = 10$ and $\Delta t = 60$, and with a convergence limit of 0.1%. If we compare the two figures, we find that the difference between the two sweeps is more noticeable when large Δt values are used. Therefore, for this difference to be minimum, small time step sizes are always preferred.

On the other hand, a lower limit was observed on the time step size beyond which the solution diverged during the global iterations. This limit depends on the initial guess of the shock shape and on how far the calculations are to proceed downstream on the body surface. As a conclusion, it is clear that the proper time step size for the present method is the smallest value which yields to a converged solution.

Comparisons of the present results with Davis and Werle, et al., references [4] and [5], are shown in figures (6) to (8). The calculated wall pressure distribution in figure (6) shows that the present results agree well with the other calculations. Figures (7) and (8) also indicate that good results for the surface skin friction coefficient and the Stanton number are obtained. In general, the agreement between the results of the different methods are remarkably good and

they essentially reproduce one another.

To complete the comparison, another test condition, with a high Reynolds number, was selected. Figure (9) shows the calculated results, together with the experimental data of Little [12], for a flow over 10° half angle hyperboloid with a Reynolds number of 63,800, free stream Mach number of 10.12, wall to stagnation temperature ratio of 0.9, and a free stream temperature of 90°R. The comparisons are remarkably good over the entire body length, specially downstream at a distance three times the nose radius.

Based on all the previous comparisons, the present method is comparable in accuracy with the other methods and is capable of predicting excellent results without the oscillatory behavior observed in reference [5] and without the stability limitation on the number of the global iterations observed in reference [10]. Few other cases were also investigated to show the applicability of the method to regions far away from the stagnation point and the capability of handling slender body configurations. Figures (10), (11) and (12) show the pressure, skin friction and Stanton number distributions over 10° and 22.5° half angle hyperboloids. The results of these test cases were obtained using a time step size of $\Delta t = 600$ and 400, and a spatial step size of $\Delta s = 0.1$ and $\Delta\eta = 0.02$. The numerical calculations were performed on Amdahl 470 V/6 computer, and the computation time was 25 minutes. With these results, it is clear that the method was not only very successful in extending the region of calculations far downstream without any numerical difficulties, but also successful in handling slender body configurations.

VI. CONCLUSIONS

A preliminary investigation has been made of the use of a coupling scheme to solve the fully viscous shock layer equations. The time relaxation technique, of reference [11], was also incorporated into the scheme to allow for the shock shape changes during the iteration process. Through application of the method to hypersonic flow past hyperboloids, it was found that the method is stable and produces results in good comparison with other methods.

The main success of the scheme lies in its ability of extending the flow field calculations far aft of the nose region without the difficulties observed with the cascading scheme of references [4] and [5]. The method is also capable of handling slender body configurations where a large portion of the flow field in the shock layer will be predominantly inviscid.

There is no difficulty in the choice of the shock initial shape and shapes parallel to the body surface with constant thickness are very adequate. However, the time step size is of fundamental importance to the solution convergence and it depends mainly on how far downstream the flow field is to be investigated.

Difficulties were encountered when the method was applied to sphere configurations and hyperboloid configurations with high Reynolds number. In these cases, variable grid size across the shock layer must be used and further studies are needed for the proper choice.

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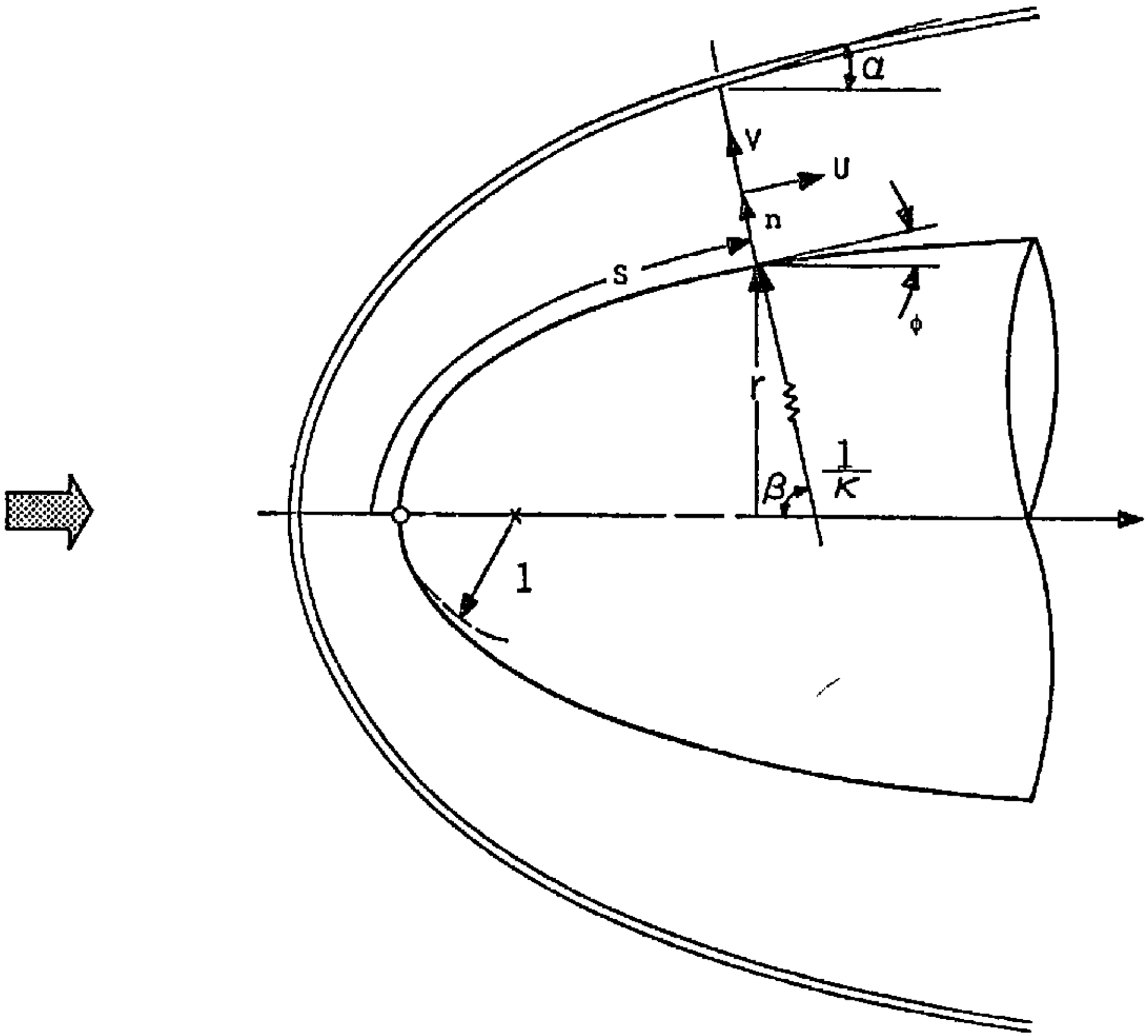

FIGURE 1. COORDINATE SYSTEM

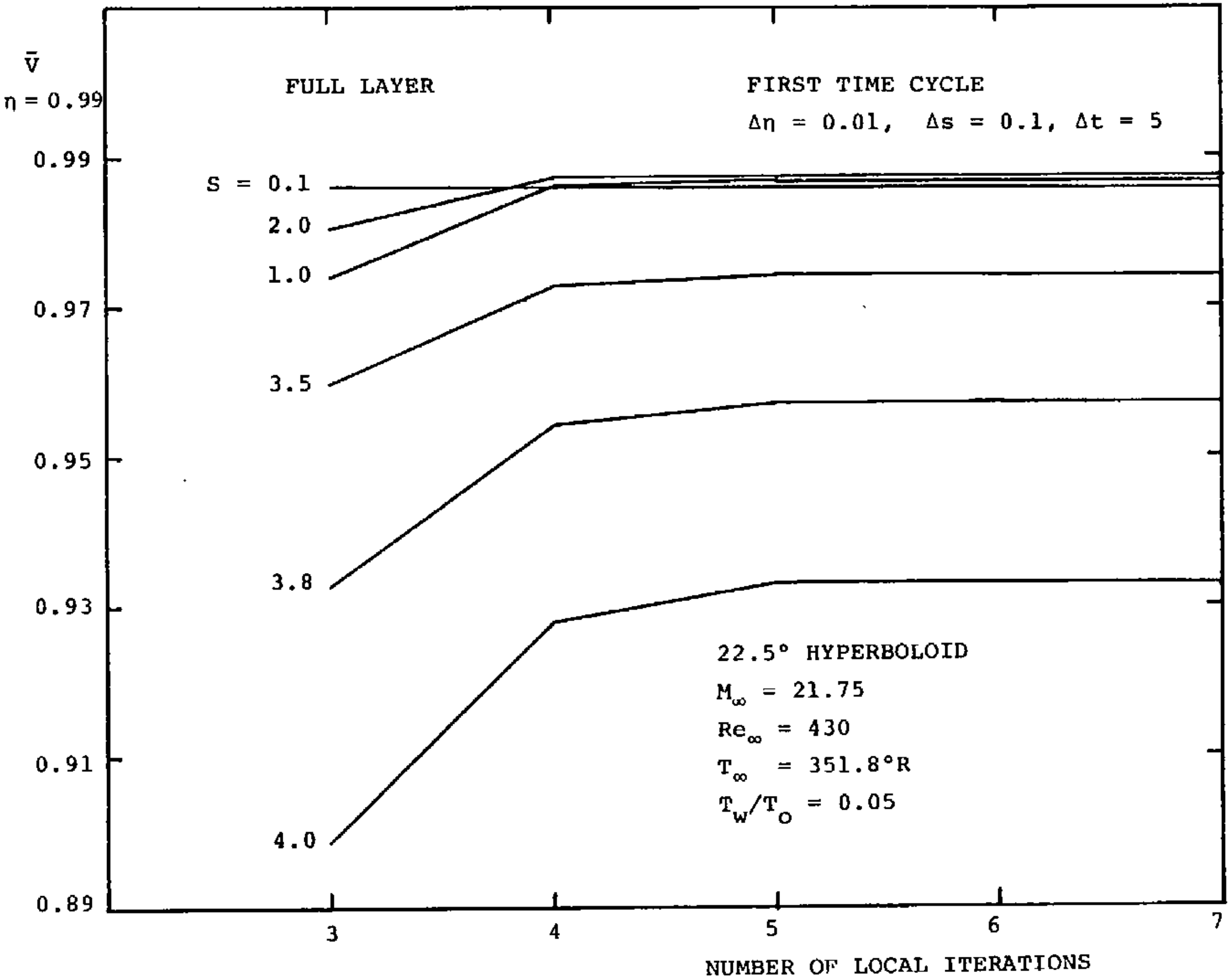

FIGURE 2. LOCAL ITERATION CONVERGENCE.

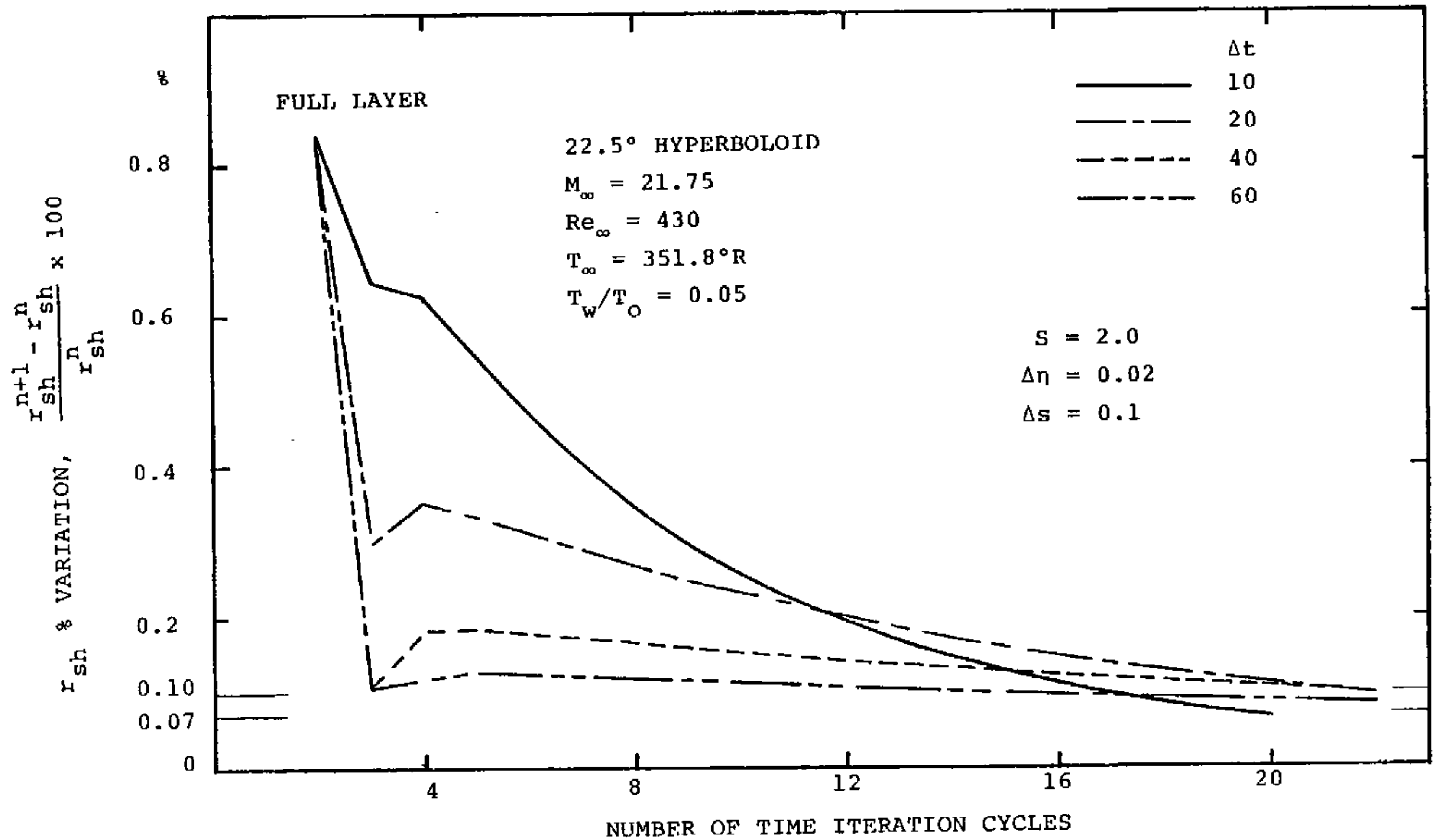

FIGURE 3. SOLUTION CONVERGENCE FOR DIFFERENT TIME STEP SIZE.

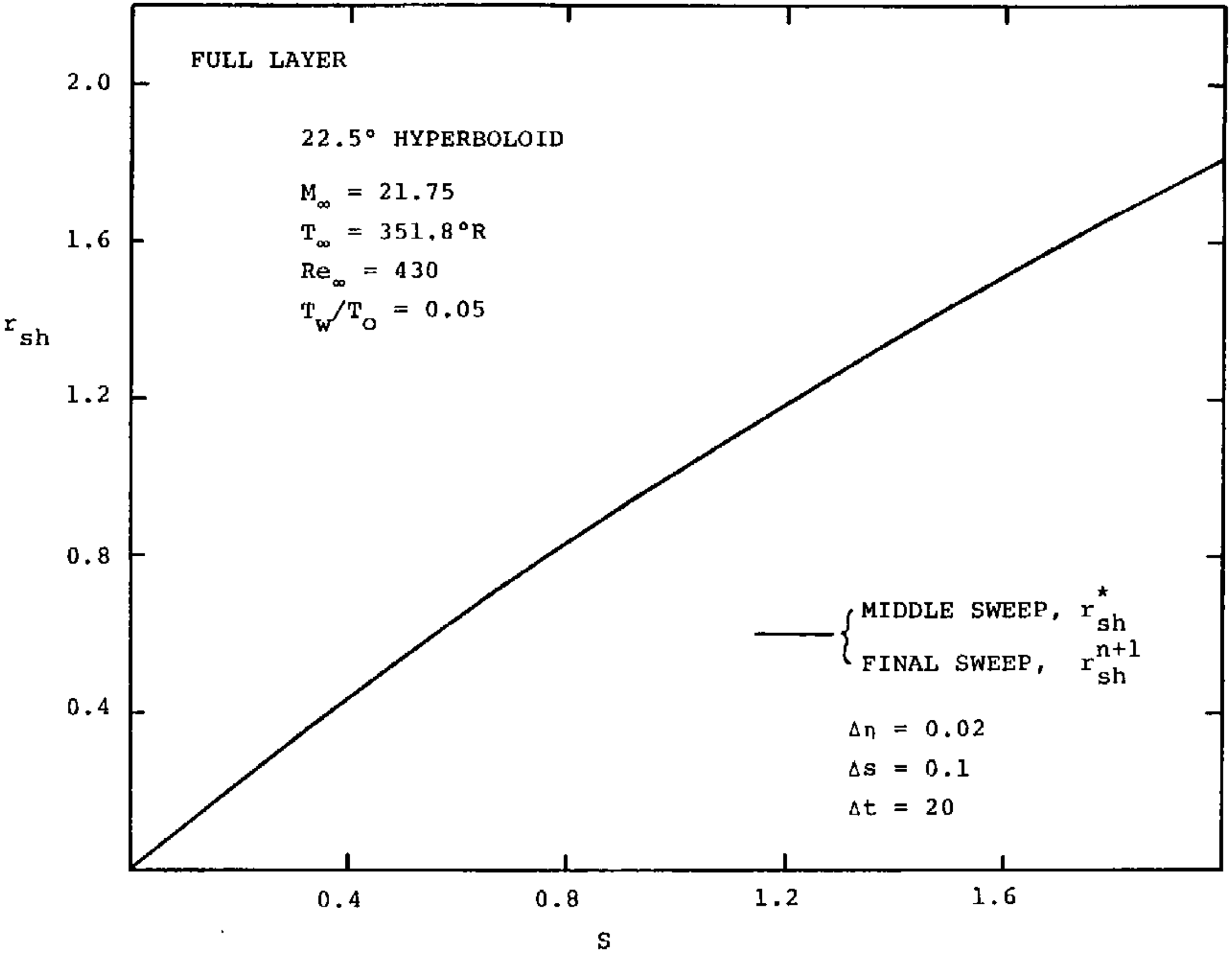

FIGURE 4. SHOCK RADIUS, $\Delta t = 20$.

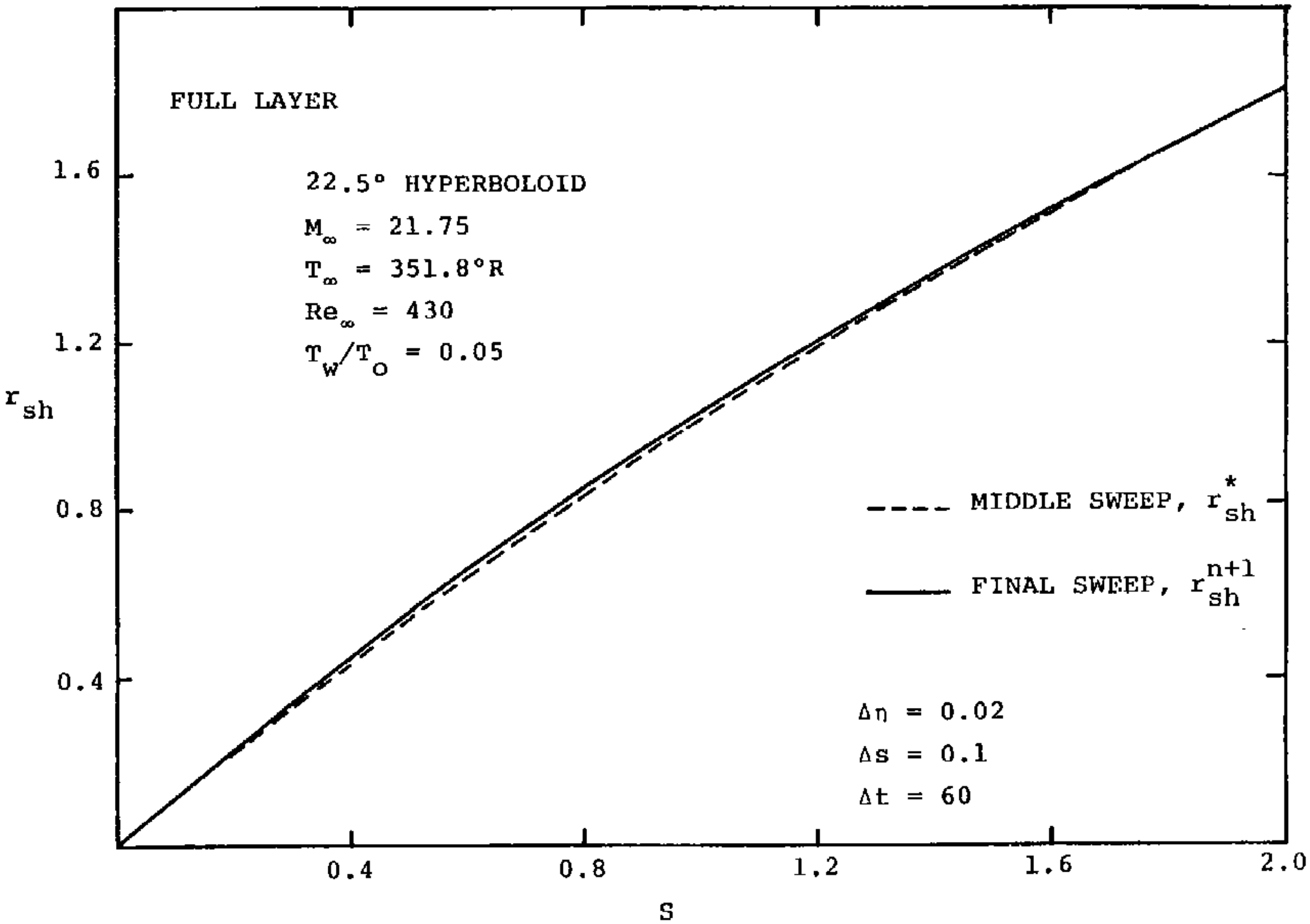

FIGURE 5. SHOCK RADIUS, Δt = 60.

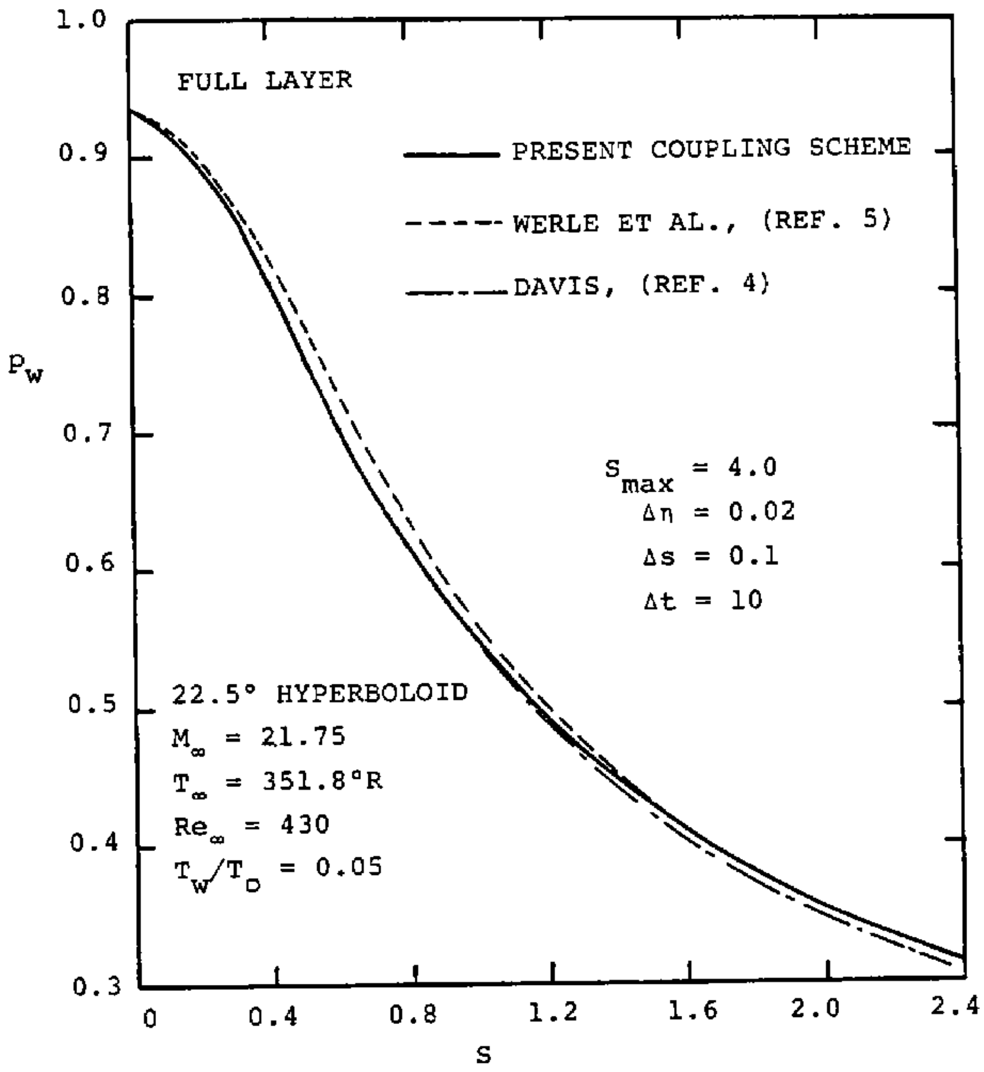

FIGURE 6. SURFACE PRESSURE DISTRIBUTION COMPARISON.

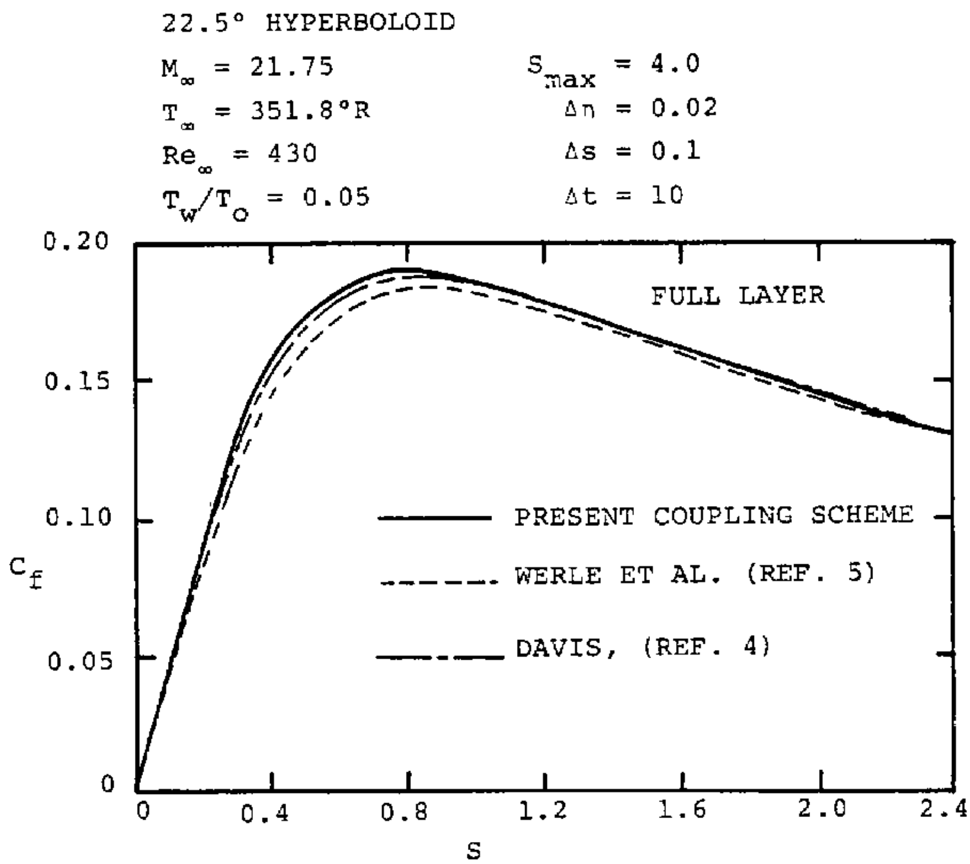

FIGURE 7. SKIN FRICTION DISTRIBUTION COMPARISON.

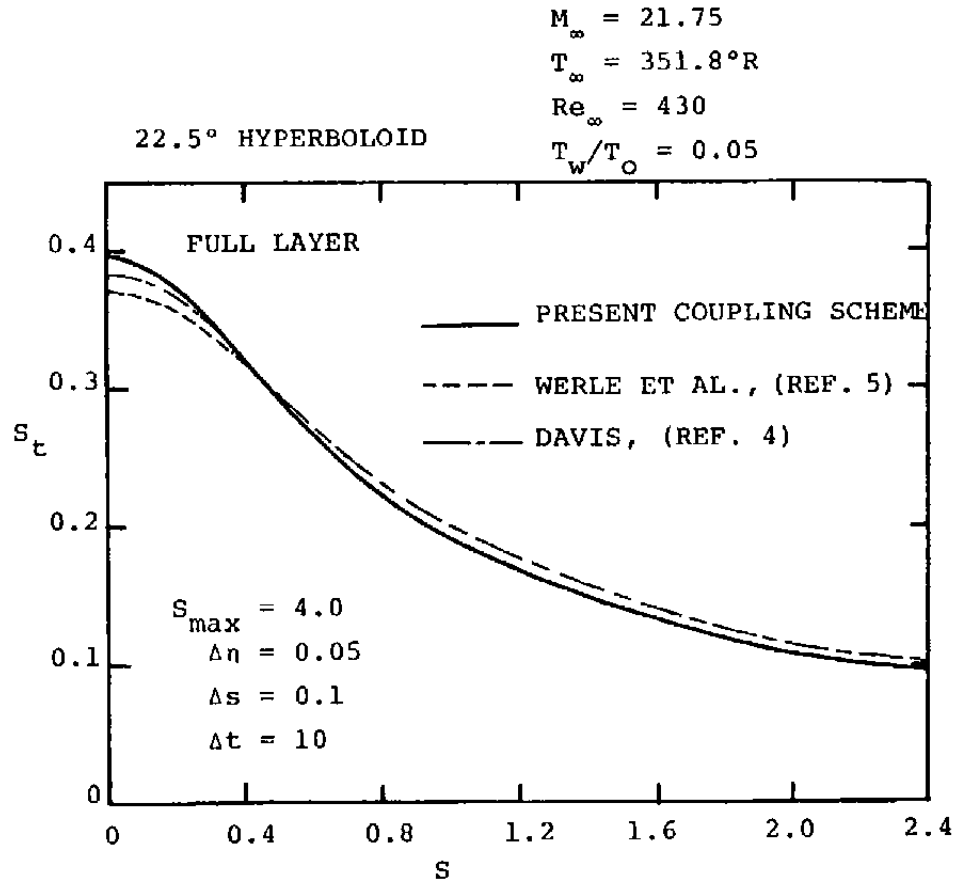

FIGURE 8. STANTON NUMBER DISTRIBUTION COMPARISON.

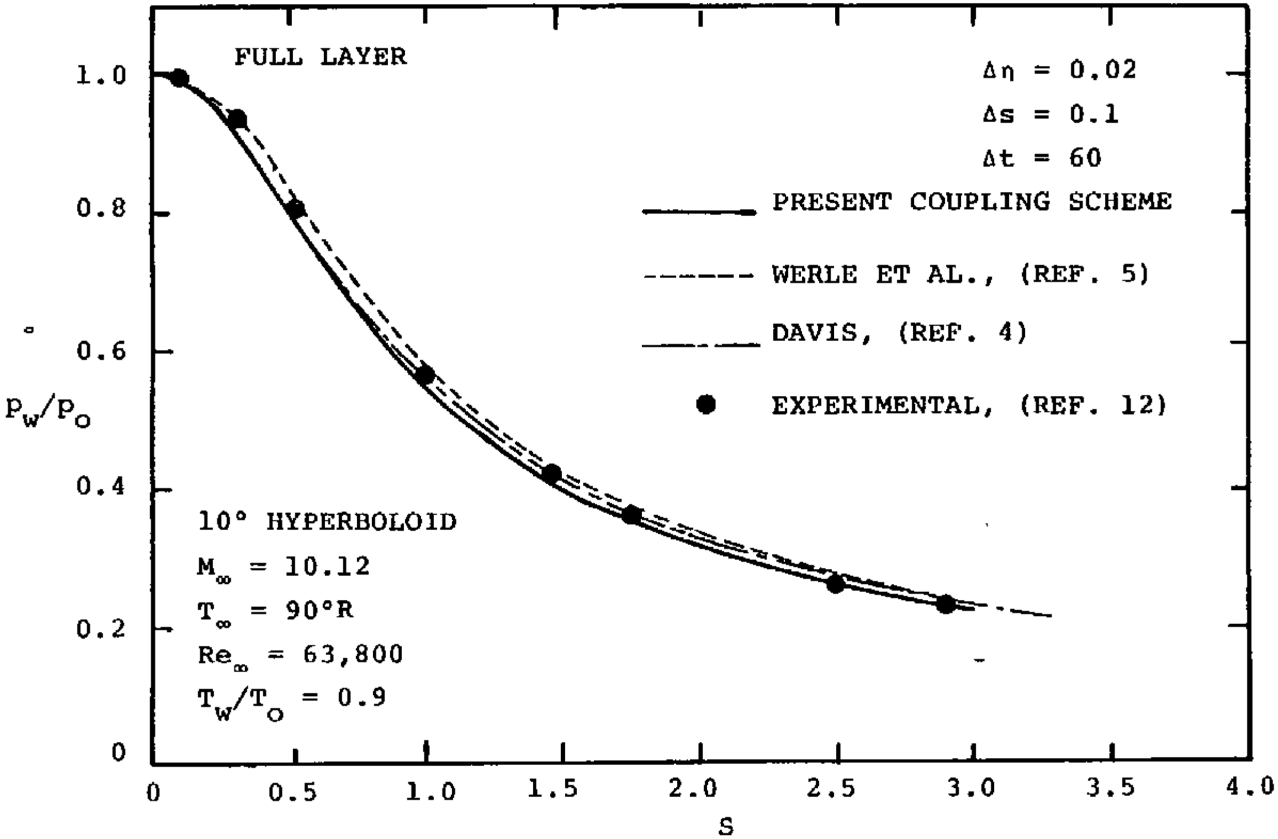

FIGURE 9. SURFACE PRESSURE DISTRIBUTION COMPARISON.

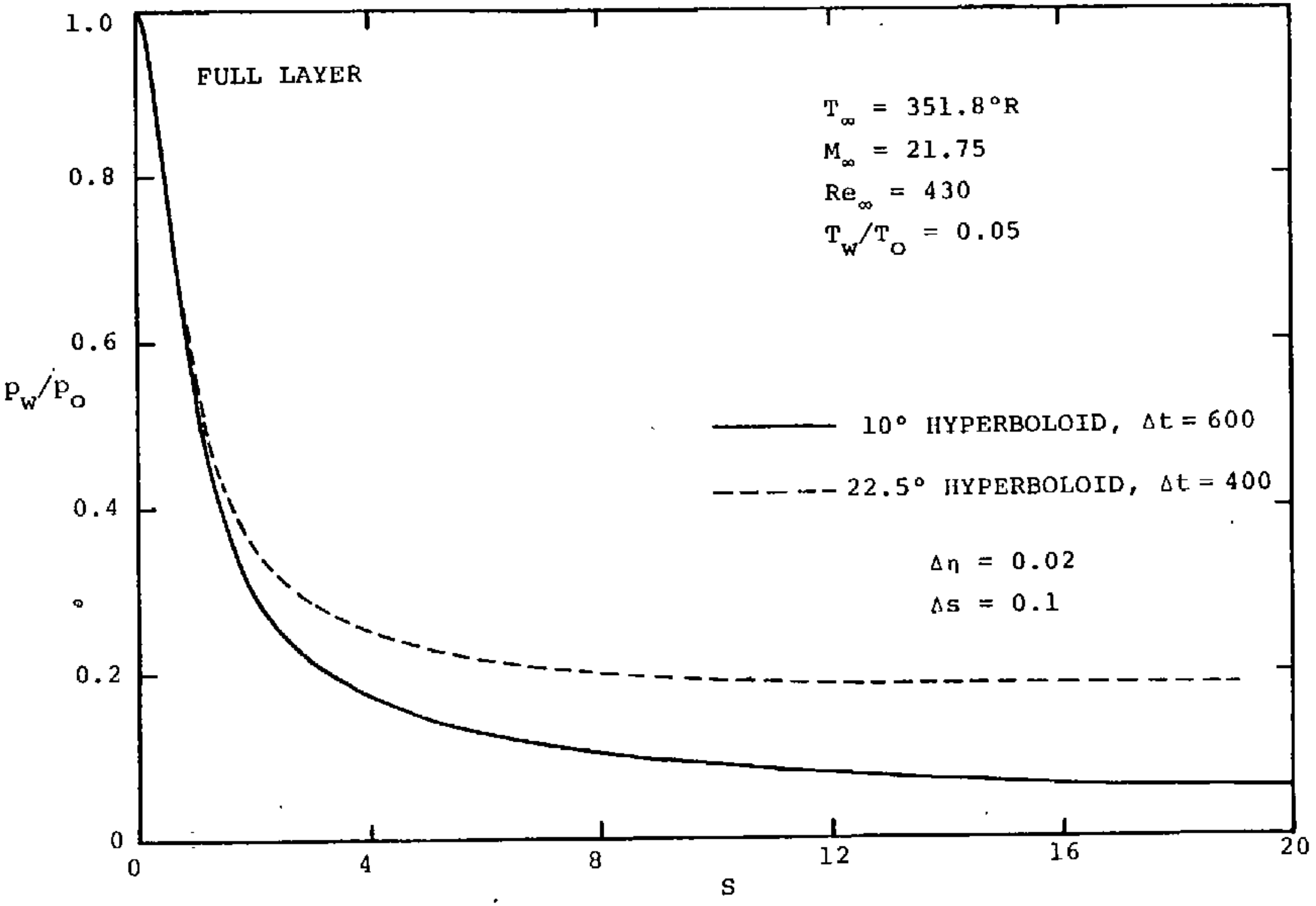

FIGURE 10. SURFACE PRESSURE DISTRIBUTION.

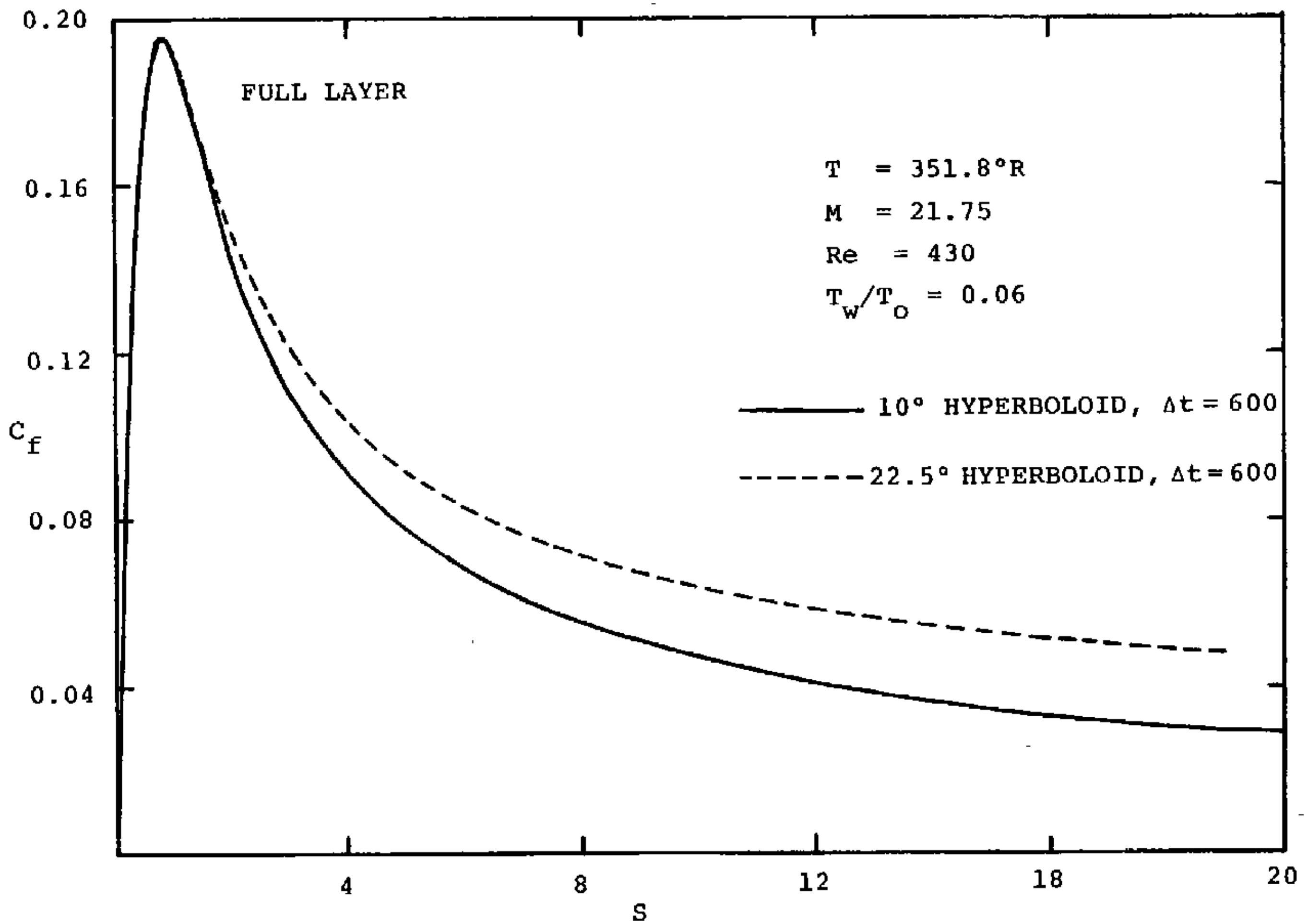

FIGURE 11. SKIN FRICTION DISTRIBUTION.

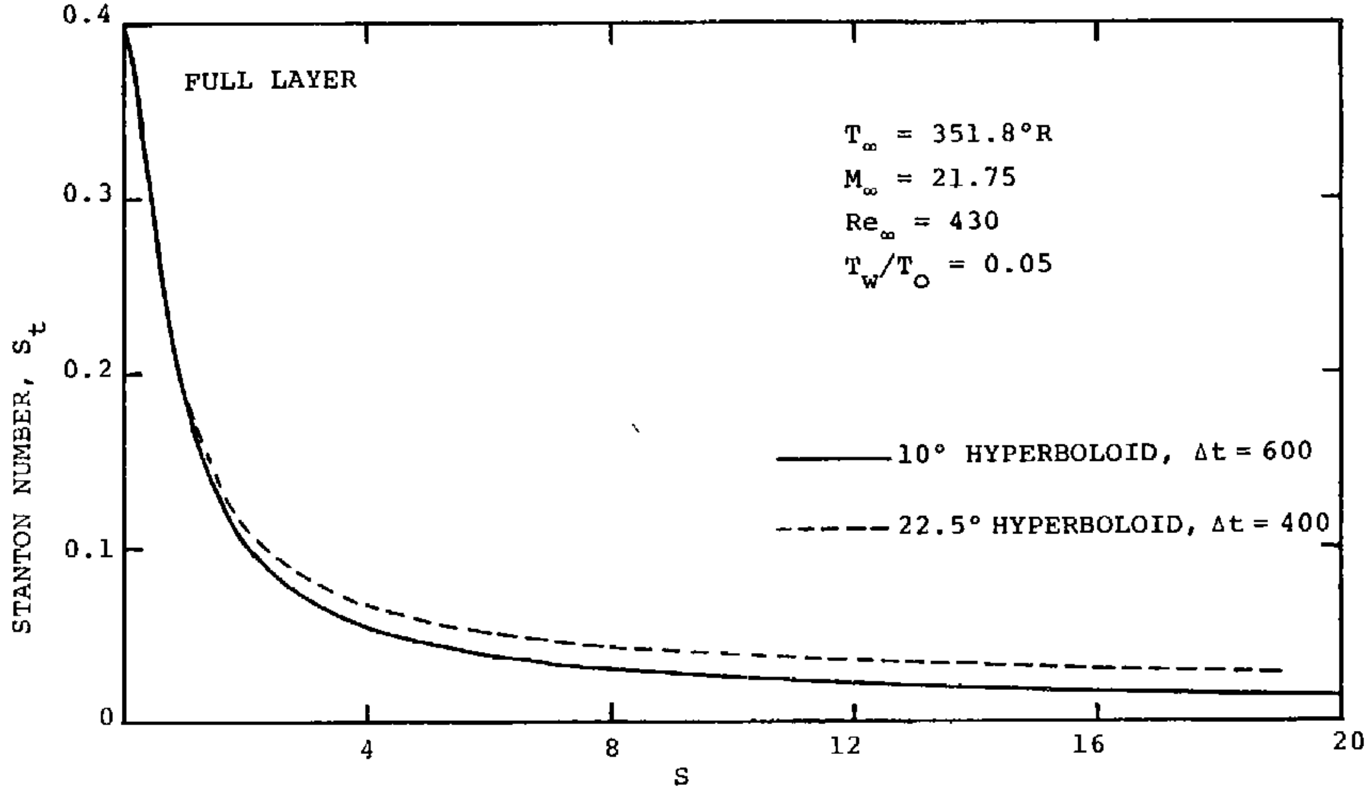

FIGURE 12. STANTON NUMBER DISTRIBUTION.

APPENDIX A

SHOCK DERIVATIVES

The shock derivatives du_{sh}/ds, dT_{sh}/ds, dp_{sh}/ds, and dv_{sh}/ds have been presented in detail in Appendix A of reference [5] and their derivation will not be repeated. For present purposes, the results are only summarized and are given by:

$$\frac{du_{sh}}{ds} = K_1 \frac{d\alpha}{ds} + K_2 \frac{d\phi}{ds} \tag{A1}$$

$$\frac{dp_{sh}}{ds} = K_3 \frac{d\alpha}{ds} \tag{A2}$$

$$\frac{dT_{sh}}{ds} = K_4 \frac{d\alpha}{ds} \tag{A3}$$

and

$$\frac{dv_{sh}}{ds} = K_5 \frac{d\alpha}{ds} + K_6 \frac{d\phi}{ds} \tag{A4}$$

where

$$K_1 = \left(\frac{\gamma-1}{2} \frac{T_{sh}}{p_{sh}} - 1\right) \sin(2\alpha - \phi) + \sin\alpha \, \sin(\alpha - \phi) \, \frac{\gamma-1}{\gamma} K_1'$$

$$K_2 = \cos\alpha \, \sin(\alpha - \phi) - \frac{\gamma-1}{\gamma} \frac{T_{sh}}{p_{sh}} \sin\alpha \, \cos(\alpha - \phi)$$

$$K_3 = \frac{2}{(\gamma+1)} \sin 2\alpha$$

$$K_4 = \frac{2\gamma}{(\gamma+1)^2} \sin 2\alpha + \frac{4}{(\gamma+1)^2 M_\infty^4} \frac{\cos\alpha}{\sin^3\alpha}$$

$$K_5 = (1 - \frac{\gamma-1}{\gamma} \frac{T_{sh}}{P_{sh}}) \cos(2\alpha - \phi) - \sin\alpha \cos(\alpha - \phi) \frac{\gamma-1}{\gamma} K_1'$$

and

$$K_6 = -\cos\alpha \cos(\alpha - \phi) - \sin\alpha \sin(\alpha - \phi) \frac{\gamma-1}{\gamma} \frac{T_{sh}}{P_{sh}}$$

where

$$K_1' = \frac{2\gamma^2 M_\infty^2}{(R_1(\gamma+1)} \sin 2\alpha + \frac{4\gamma}{(\gamma+1) M_\infty^2 R_1} \frac{\cos\alpha}{\sin^3\alpha} - \frac{4\gamma^3 M_\infty^4 \sin 2\alpha \sin^2\alpha}{R_1^2 (\gamma+1)}$$

$$- \frac{2\gamma M_\infty^2 \sin 2\alpha}{R_1^2} [\frac{\gamma(\gamma+1)}{\gamma-1} - \frac{2\gamma(\gamma-1)}{\gamma+1}] + \frac{4\gamma^2 \sin 2\alpha}{(\gamma+1) R_1^2 \sin^2\alpha}$$

and

$$R_1 = [2\gamma M_\infty^2 \sin^2\alpha - (\gamma-1)]$$

APPENDIX B

SHOCK SLOPE DERIVATIVE

The shock slope derivative is derived in this appendix, for use, together with shock derivatives of Appendix A, in the viscous shock layer solution.

In the spatial coordinate system the shock angle, α, is written as, (Figure 1)

$$\alpha = \tan^{-1}\left(\frac{dR}{dx_{sh}}\right) \tag{B1}$$

where $R = y_B + n_{sh}\cos\phi$ and $x_{sh} = x_B - n_{sh}\sin\phi$ (B2)

Hence the derivative $d\alpha/ds$ is evaluated as,

$$\frac{d\alpha}{ds} = \frac{1}{\left[1 + \left(\frac{dR}{dx_{sh}}\right)^2\right]} \frac{d^2R}{dx_{sh}^2} \frac{dx_{sh}}{ds} \tag{B3}$$

Note that

$$\frac{dx_{sh}}{ds} = \cos\phi(1+\kappa n_{sh}) - n_{sh}' \sin\phi \tag{B4}$$

and

$$\frac{dn_{sh}}{ds} = (1+\kappa n_{sh}) \tan(\alpha-\phi) \tag{B5}$$

combining (B4) and (B5) yields

$$\frac{dx_{sh}}{ds} = (1+\kappa n_{sh}) \frac{\cos\alpha}{\cos(\alpha-\phi)} \tag{B6}$$

Hence the derivative d^2x_{sh}/ds^2 is evaluated as,

$$\frac{d^2x_{sh}}{ds^2} = (\kappa' n_{sh} + \kappa n'_{sh}) \frac{\cos\alpha}{\cos(\alpha-\phi)} + (1+\kappa n_{sh}) \left[- \frac{\sin\alpha}{\cos(\alpha-\phi)} \frac{d\alpha}{ds} + \frac{\cos\alpha}{\cos^2(\alpha-\phi)} \sin(\alpha-\phi) \left(\frac{d\alpha}{ds} + \kappa\right)\right] \tag{B7}$$

substituting (B6) in (B3) and noting that $dR/dx_{sh} = \tan\alpha$ yields after certain manipulations,

$$\frac{d\alpha}{ds} = (1+\kappa n_{sh}) \frac{\cos^3\alpha}{\cos(\alpha-\phi)} \frac{d^2R}{dx_{sh}^2} \tag{B8}$$

It is to be also noted that,

$$\tan\alpha = \frac{dR}{dx_{sh}} = \frac{dR/ds}{dx_{sh}/ds} \tag{B9}$$

Hence the second derivative, d^2R/dx_{sh}^2, can be shown to be

$$\frac{d^2R}{dx_{sh}^2} = \frac{d^2R/ds^2}{(dx_{sh}/ds)^2} - \frac{d^2x_{sh}/ds^2 \; dR/ds}{(dx_{sh}/ds)^3} \tag{B10}$$

Substituting for dx_{sh}/ds from (B6), d^2x_{sh}/ds^2 from (B7), and dR/ds from

$$\frac{dR}{ds} = (1+\kappa n_s) \frac{\sin\alpha}{\cos(\alpha-\phi)}$$

and then evaluating (B8) yields after proper manipulations,

$$\frac{d\alpha}{ds} = \frac{d^2R}{ds^2} \left[\frac{\cos^2(\alpha-\phi)}{(1+\kappa n_{sh})\cos\phi}\right] - \frac{dR}{ds} \left[\frac{\kappa\sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_{sh})} + \frac{n_{sh}\kappa'\cos^2(\alpha-\phi)}{\cos\phi(1+\kappa n_{sh})^2}\right] \tag{B11}$$

APPENDIX C

ADI FORMULATION OF S-MOMENTUM EQUATION

The s-momentum equation is written in the following normalized form

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \alpha_1 \frac{\partial \bar{u}}{\partial \eta} + \alpha_2 \bar{u} + \alpha_3 + \alpha_4 \frac{\partial \bar{u}}{\partial \xi} = 0 \qquad \text{(C1)}$$

where α_1, α_2, α_3 and α_4 are defined by equations (5a-d). Note that u'_{sh} and p'_{sh} appear in the coefficients α_2 and α_3. Upon using equations (A1), (A2) and (B11) into equation (C1) and after certain manipulations the following equation is achieved, (details are given in reference 11),

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} + \beta_2 \frac{d^2 R}{ds^2} + \beta_3 \frac{dR}{ds} + \beta_4 + \beta_5 \frac{\partial \bar{u}}{\partial \xi} = 0 \qquad \text{(C2)}$$

where

$$\begin{aligned} \beta_1 &= \alpha_1 \\ \beta_2 &= \gamma_2 \bar{u} + \gamma_5 \\ \beta_3 &= \gamma_3 \bar{u} + \gamma_6 \\ \beta_4 &= \gamma_4 \bar{u} + \gamma_7 \\ \beta_5 &= \alpha_4 \end{aligned} \qquad \text{(C3)}$$

and γ_2, γ_3, γ_4, γ_5, γ_6 and γ_7 are given by

$$\gamma_2 = -A\, K_1 \frac{\cos^2(\alpha-\phi)}{(1+\kappa n_{sh})\cos\phi}$$

$$\gamma_3 = +A\, K_1 \left[\frac{\kappa \sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_{sh})} + \frac{n_{sh}\kappa' \cos^2(\alpha-\phi)}{\cos\phi(1+\kappa n_{sh})^2}\right]$$

$$\gamma_4 = +A\kappa\ K_2 + B + C + D$$

$$\gamma_5 = A_1 \frac{\bar{p}}{p_{sh}} K_3 \frac{\cos^2(\alpha-\phi)}{(1+\kappa n_{sh})\cos\phi}$$

$$\gamma_6 = -A_1 \frac{\bar{p}}{p_{sh}} K_3 \left[\frac{\kappa\sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_{sh})} + \frac{n_s\ \kappa'\ \cos^2(\alpha-\phi)}{\cos\phi(1+\kappa n_{sh})^2}\right.$$

$$\gamma_7 = A_1 \left(\bar{p}_\xi - \frac{n'_{sh}}{n_{sh}}\ \eta\ \bar{p}_\eta\right) \qquad \text{(C4)}$$

where

$$A = \frac{\rho_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}}\ \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta}\ \frac{\bar{\rho}\bar{u}}{\bar{\mu}}$$

$$B = -\frac{\rho_{sh} v_{sh} n_{sh}}{\varepsilon^2\ \mu_{sh}}\ \frac{n_{sh}}{1+\kappa n_{sh}\eta}\ \frac{\bar{\rho}\bar{v}}{\bar{\mu}}$$

$$C = -\frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta}\ \frac{\bar{\mu}\bar{n}}{\bar{\mu}}$$

$$D = -\left(\frac{\kappa n_{sh}}{(1+\kappa n_{sh}\eta)} + \frac{\cos\phi\ n_{sh}}{r+n_{sh}\cos\phi\eta}\right) \frac{\kappa n_{sh}}{(1+\kappa n_{sh}\eta)}$$

$$A_1 = -\frac{p_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}}\ \frac{n_{sh}}{(1+\kappa n_{sh}\eta)}\ \frac{1}{\bar{\mu}\ u_{sh}} \qquad \text{(C5)}$$

It can be shown by proper substitution that

$$\frac{\beta_3}{\beta_2} = -\left[2\kappa\ \tan(\alpha-\phi) + \frac{n_{sh}\kappa'}{1+\kappa n_{sh}}\right] \qquad \text{(C6)}$$

APPENDIX D

GOVERNING EQUATIONS LINEARIZATION

s-Momentum Equation:

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \alpha_1 \left(\frac{\partial \bar{u}}{\partial \eta}\right) + \alpha_2 \bar{u} + \alpha_3 + \alpha_4 \left(\frac{\partial \bar{u}}{\partial \xi}\right) = 0 \tag{D1}$$

where α_1, α_2, α_3 and α_4 are given by equations (5a-d), and they are rewritten as,

$$\alpha_1 = \alpha_{11}\, \bar{\rho}\bar{u} + \alpha_{12}\, \bar{\rho}\bar{v} + \alpha_{13}$$

$$\alpha_2 = \alpha_{21}\, \bar{\rho}\bar{u} + \alpha_{22}\, \bar{\rho}\bar{v} + \alpha_{23}$$

$$\alpha_3 = \alpha_{31}\left(\bar{p}_\xi - \frac{n'_{sh}}{n_{sh}}\, \eta\, \bar{p}_\eta + \frac{p'_{sh}}{p_{sh}}\, \bar{p}\right)$$

$$\alpha_4 = \alpha_{41}\, \bar{\rho}\bar{u} \tag{D2}$$

and

$$\alpha_{11} = \frac{\rho_{sh} u_{sh} n'_{sh}}{\varepsilon^2 \mu_{sh}}\, \frac{n_{sh}}{1+\kappa n_{sh}\eta}\, \frac{\eta}{\bar{\mu}}$$

$$\alpha_{12} = -\frac{\rho_{sh} v_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}}\, \frac{1}{\bar{\mu}}$$

$$\alpha_{13} = \frac{\bar{\mu}_\eta}{\bar{\mu}} + \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi\; n_{sh}}{r+n_{sh}\,\eta\cos\phi}$$

$$\alpha_{21} = -\frac{\rho_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}}\, \frac{n_{sh}}{1+\kappa n_{sh}}\, \frac{u'_{sh}}{\bar{\mu}}$$

$$\alpha_{22} = -\frac{\rho_{sh} v_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}} \frac{1}{\bar{\mu}}$$

$$\alpha_{23} = -\kappa \frac{n_{sh}}{1+\kappa n_{sh}\eta} \left(\frac{\bar{\mu}_\eta}{\bar{\mu}} + \kappa \frac{n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi \; n_{sh}}{r+n_{sh}\eta\cos\phi}\right)$$

$$\alpha_{31} = -\frac{p_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{1}{\bar{\mu}} \frac{1}{u_{sh}}$$

$$\alpha_{41} = -\frac{\rho_{sh} u_{sh} n_{sh}}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{1}{\bar{\mu}} \tag{D3}$$

Using the linearization formulas given by equations (22a-c), equation (D1) could be written as,

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \left(\alpha_1^o \frac{\partial \bar{u}}{\partial \eta} + \alpha_2^o \bar{u} + \alpha_4^o \frac{\partial \bar{u}}{\partial \xi}\right)$$
$$+ \left(\frac{\partial \bar{u}^o}{\partial \eta} \alpha_1 + \bar{u}^o \alpha_2 + \frac{\partial \bar{u}^o}{\partial \xi} \alpha_4\right)$$
$$- \left(\alpha_1^o \frac{\partial \bar{u}^o}{\partial \eta} + \alpha_2^o \bar{u}^o + \alpha_4^o \frac{\partial \bar{u}^o}{\partial \xi}\right) = 0 \tag{D4}$$

where the quantities with superscript ^{o} are evaluated from either the previous iteration, or the previous ξ step. Upon using equations (D2) and (D3), substituting for the density from the equation of state $\bar{p} = \bar{\rho}\bar{t}$, and using the linearization formulas (22a-c), equation (D4) can be written in the following form,

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + (a_1 \bar{u}_\xi + a_3 \bar{p}_\xi) + (b_1 \bar{u}_\eta + b_3 \bar{p}_\eta)$$
$$+ (c_1 \bar{u} + c_2 \bar{v} + c_3 \bar{p} + c_4 \bar{t}) + d = 0 \tag{D5}$$

where the coefficients a_1, a_3, ... etc., are given by the following forms,

$$a_1 = - \gamma_1 \gamma_2 \frac{u_{sh}}{v_{sh}} \frac{1}{\bar{\mu}} \bar{\rho}^o \bar{u}^o$$

$$a_3 = - \gamma_1 \gamma_2 \frac{p_{sh}}{\rho_{sh} u_{sh} v_{sh}} \frac{1}{\bar{\mu}}$$

$$b_1 = \gamma_1 \left[\gamma_2 \frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \frac{n}{\bar{\mu}} \bar{\rho}^o \bar{u}^o - \frac{\bar{\rho}^o \bar{v}^o}{\bar{\mu}}\right] + \frac{\bar{\mu}_n}{\bar{\mu}} + \kappa \gamma_2 + \gamma_3$$

$$b_3 = \gamma_1 \gamma_2 \frac{p_{sh}}{\rho_{sh} u_{sh} n_{sh}} \frac{n'_{sh}}{n_{sh}} \frac{n}{\bar{\mu}}$$

$$c_1 = - \gamma_2 \left[\gamma_1 \bar{\rho}^o \left(2 \frac{u'_{sh}}{v_{sh}} \frac{\bar{u}^o}{\bar{\mu}} + \kappa \frac{\bar{v}^o}{\bar{\mu}} - \frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \frac{n}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial n} \right.\right.$$

$$\left.\left. + \frac{u_{sh}}{v_{sh}} \frac{1}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial \xi}\right) + \kappa \left(\frac{\bar{\mu}_n}{\bar{\mu}} + \kappa \gamma_2 + \gamma_3\right)\right]$$

$$c_2 = - \gamma_1 \bar{\rho}^o \left(\frac{1}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial \eta} + \kappa \gamma_2 \frac{\bar{u}^o}{\bar{\mu}}\right)$$

$$c_4 = - \gamma_1 \frac{\bar{\rho}^o}{\bar{t}^o} \left[\gamma_2 \left(\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \frac{n}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial r} - \frac{u'_{sh}}{v_{sh}} \frac{1}{\bar{\mu}} \bar{u}^o - \frac{u_{sh}}{v_{sh}} \frac{1}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial \xi} \right.\right.$$

$$\left.\left. - \kappa \frac{\bar{v}^o}{\bar{\mu}}\right) \bar{u}^o - \frac{1}{\bar{\mu}} \frac{\partial \bar{u}^o}{\partial n} \bar{v}^o\right]$$

$$c_3 = - \frac{c_4}{\bar{\rho}^o} - \gamma_1 \gamma_2 \frac{p'_{sh}}{\rho_{sh} u_{sh} v_{sh}} \frac{1}{\bar{\mu}}$$

and

$$d = \gamma_1\{\gamma_2[(-\frac{u_{sh}'}{v_{sh}}\frac{n_{sh}'}{n_{sh}}\eta\frac{\partial\bar{u}^o}{\partial\eta} + \frac{u_{sh}'}{v_{sh}}\bar{u}^o + \frac{u_{sh}}{v_{sh}}\frac{\partial\bar{u}^o}{\partial\xi})\frac{\bar{\rho}^o u^o}{\bar{\mu}}$$

$$+ \kappa\frac{\bar{\rho}^o\bar{u}^o\bar{v}^o}{\bar{\mu}}] + \frac{1}{\bar{\mu}}\bar{\rho}^o\bar{v}^o\frac{\partial\bar{u}^o}{\partial\eta}\} \quad \text{(D6)}$$

where

$$\gamma_1 = \frac{\rho_{sh}v_{sh}n_{sh}}{\varepsilon^2\mu_{sh}}$$

$$\gamma_2 = \frac{n_{sh}}{1+\kappa n_{sh}\eta}$$

$$\gamma_3 = \frac{n_{sh}\cos\phi}{r+n_{sh}\eta\cos\phi} \quad \text{(D7)}$$

Energy Equation:

The energy equation is given by equation (6) and is written in the form,

$$\frac{\partial^2\bar{t}}{\partial\eta^2} + \alpha_1\frac{\partial\bar{t}}{\partial\eta} + \alpha_2\bar{t} + \alpha_3 + \alpha_4\frac{\partial\bar{t}}{\partial\xi} = 0 \quad \text{(D8)}$$

where the coefficients α_1, α_2, α_3 and α_4 are given by equations (6a-d), and they are rewritten as,

$$\alpha_1 = \alpha_{11}\bar{\rho}\bar{u} + \alpha_{12}\bar{\rho}\bar{v} + \alpha_{13}$$

$$\alpha_2 = \alpha_{21}\bar{\rho}\bar{u}$$

$$\alpha_3 = \alpha_{31} \left(\bar{p}_\xi - \frac{n'_{sh}}{n_{sh}} \eta \bar{p}_\eta + \frac{p'_{sh}}{p_{sh}} \bar{p}\right) \bar{u} + \alpha_{32} \bar{v} \bar{p}_\eta$$

$$+ \alpha_{33} \left(\bar{u}_\eta - \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} \bar{u}\right)^2$$

$$\alpha_4 = \alpha_{41} \bar{\rho} \bar{u} \tag{D9}$$

and

$$\alpha_{11} = \frac{\rho_{sh} u_{sh} n'_{sh} \sigma}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}} \frac{\eta}{\bar{\mu}}$$

$$\alpha_{12} = - \frac{\rho_{sh} v_{sh} n_{sh} \sigma}{\varepsilon^2 \mu_{sh}} \frac{1}{\bar{\mu}}$$

$$\alpha_{13} = \frac{\bar{\mu}_\eta}{\bar{\mu}} + \frac{\kappa n_{sh}}{1+\kappa n_{sh}\eta} + \frac{\cos\phi\, n_{sh}}{r+n_{sh}\eta\cos\phi}$$

$$\alpha_{21} = - \frac{\rho_{sh} u_{sh} \sigma}{\varepsilon^2 \mu_{sh} T_{sh}} \frac{n^2_{sh}}{1+\kappa n_{sh}\eta} \frac{T'_{sh}}{\bar{\mu}}$$

$$\alpha_{31} = \frac{p_{sh} u_{sh} n_{sh} \sigma}{\varepsilon^2 \mu_{sh} T_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{1}{\bar{\mu}}$$

$$\alpha_{32} = \frac{p_{sh} v_{sh} n_{sh} \sigma}{\varepsilon^2 \mu_{sh} T_{sh}} \frac{1}{\bar{\mu}}$$

$$\alpha_{33} = \frac{u^2_{sh} \sigma}{T_{sh}}$$

$$\alpha_{41} = - \frac{\rho_{sh} u_{sh} n_{sh} \sigma}{\varepsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}\eta} \frac{1}{\bar{\mu}} \tag{D10}$$

The linearization of equation (D8), results in the following equation

$$\frac{\partial^2 \bar{t}}{\partial \eta^2} + (\alpha_1^o \frac{\partial \bar{t}}{\partial \eta} + \alpha_2^o \bar{t} + \alpha_4^o \frac{\partial \bar{t}}{\partial \xi})$$

$$+ (\frac{\partial \bar{t}^o}{\partial \eta} \alpha_1 + \bar{t}^o \alpha_2 + \alpha_3 + \frac{\partial \bar{t}^o}{\partial \xi} \alpha_4)$$

$$- (\alpha_1^o \frac{\partial \bar{t}^o}{\partial \eta} + \alpha_2^o \bar{t}^o + \alpha_4^o \frac{\partial \bar{t}^o}{\partial \xi}) = 0 \qquad \text{(D11)}$$

Upon using equations (D9) and (D10), substituting for the density from the equation of state $\bar{p} = \bar{\rho}\bar{t}$, and using the linearization formulas (22a-c), equation (D11) can be written in the following form

$$\frac{\partial^2 \bar{t}}{\partial \eta^2} + (a_3 \bar{p}_\xi + a_4 \bar{t}_\xi) + (b_1 \bar{u}_\eta + b_3 \bar{p}_\eta + b_4 \bar{t}_\eta)$$

$$+ (c_1 \bar{u} + c_2 \bar{v} + c_3 \bar{p} + c_4 \bar{t}) + d = 0 \qquad \text{(D12)}$$

where the coefficients a_3, a_4,... etc., are given below

$$a_3 = \gamma_1 \gamma_2 \frac{u_{sh}}{v_{sh}} \frac{p_{sh}}{\rho_{sh} T_{sh}} \frac{\sigma}{\bar{\mu}} \bar{u}^o$$

$$a_4 = - \gamma_1 \gamma_2 \frac{u_{sh}}{v_{sh}} \frac{\sigma}{\bar{\mu}} \bar{\rho}^o \bar{u}^o$$

$$b_1 = 2 \frac{u_{sh}^2 \sigma}{T_{sh}} (\bar{u}_\eta^o - \kappa \gamma_2 \bar{u}^o)$$

$$b_3 = \gamma_1 \frac{\sigma}{\bar{\mu}} \frac{p_{sh}}{\rho_{sh} T_{sh}} (- \frac{u_{sh}}{v_{sh}} \gamma_2 \frac{n'_{sh}}{n_{sh}} \eta \bar{u}^o + \bar{v}^o)$$

$$b_4 = \gamma_1 \left(\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \gamma_2 \frac{\sigma}{\bar{\mu}} \eta \bar{\rho}^o \bar{u}^o - \frac{\sigma}{\bar{\mu}} \bar{\rho}^o \bar{v}^o\right)$$

$$+ \frac{\bar{u}_\eta}{\bar{\mu}} + \kappa\gamma_2 + \gamma_3$$

$$c_1 = \gamma_2 \sigma \{ \gamma_1 [\bar{\rho}^o (\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \eta \frac{\partial \bar{t}^o}{\partial \eta} - \frac{u_{sh}}{v_{sh}} \frac{T'_{sh}}{T_{sh}} \bar{t}^o$$

$$- \frac{u_{sh}}{v_{sh}} \frac{\partial \bar{t}^o}{\partial \xi}) \frac{1}{\bar{\mu}} + \frac{u_{sh}}{v_{sh}} \frac{p_{sh}}{\rho_{sh} T_{sh}} \frac{1}{\bar{\mu}} (\bar{p}^o_\xi - \frac{n'_{sh}}{n_{sh}} \eta \bar{p}^o_\eta$$

$$+ \frac{p'_{sh}}{p_{sh}} \bar{p}^o)] - 2\kappa \frac{u^2_{sh}}{T_{sh}} (\bar{u}^o_\eta - \kappa\gamma_2 \bar{u}^o) \}$$

$$c_2 = \gamma_1 \frac{\sigma}{\bar{\mu}} \left(- \frac{\bar{p}^o}{\bar{t}^o} \frac{\partial \bar{t}^o}{\partial \eta} + \frac{p_{sh}}{\rho_{sh} T_{sh}} \bar{p}^o_\eta\right)$$

$$c_3 = \gamma_1 \frac{\sigma}{\bar{\mu}} \{ \gamma_2 \bar{u}^o [(\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \eta \frac{\partial \bar{t}^o}{\partial \eta} - \frac{u_{sh}}{v_{sh}} \frac{T'_{sh}}{T_{sh}} \bar{t}^o$$

$$- \frac{u_{sh}}{v_{sh}} \frac{\partial \bar{t}^o}{\partial \xi}) \frac{1}{\bar{t}^o} + \frac{u_{sh}}{v_{sh}} \frac{p'_{sh}}{\rho_{sh} T_{sh}}] - \frac{\bar{v}^o}{\bar{t}^o} \frac{\partial \bar{t}^o}{\partial \eta} \}$$

$$c_4 = - \gamma_1 \sigma \{ \gamma_2 \bar{\rho}^o \bar{u}^o (\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \eta \frac{\partial \bar{t}^o}{\partial \eta} - \frac{u_{sh}}{v_{sh}} \frac{\partial \bar{t}^o}{\partial \xi})$$

$$- \bar{\rho}^o \bar{v}^o \frac{\partial \bar{t}^o}{\partial \eta} \} \frac{1}{\bar{\mu} \bar{t}^o}$$

and

$$d = - \gamma_1 \frac{\sigma}{\bar{\mu}} \{ \gamma_2 [\bar{\rho}^o \bar{u}^o (\frac{u_{sh}}{v_{sh}} \frac{n'_{sh}}{n_{sh}} \eta \frac{\partial \bar{t}^o}{\partial \eta} - \frac{u_{sh}}{v_{sh}} \frac{T'_{sh}}{T_{sh}} \bar{t}^o$$

$$- \frac{u_{sh}}{v_{sh}} \frac{\partial \bar{t}^o}{\partial \xi}) + \frac{u_{sh}}{v_{sh}} \frac{p_{sh}}{{}_{sh} T_{sh}} \bar{u}^o (\bar{p}^o_\xi - \frac{n'_{sh}}{n_{sh}} \eta \bar{p}^o_\eta + \frac{p'_{sh}}{p_{sh}} \bar{p}^o)] -$$

$$- \bar{\rho}^{o} \bar{v}^{o} \frac{\partial \bar{t}^{o}}{\partial \eta} + \frac{p_{sh}}{\rho_{sh} T_{sh}} \bar{v}^{o} \bar{p}^{o}_{\eta} \} - \frac{u^2_{sh} \sigma}{T_{sh}} (\bar{u}^{o}_{\eta} - \kappa \gamma_2 \bar{u}^{o})^2 \qquad (D13)$$

where γ_1, γ_2 and γ_3 are given by equations (D7).

n-Momentum Equation:

The normal momentum equation is given by equation (8) and is rewritten in the following form

$$\frac{\bar{\rho}\bar{u}}{1+\kappa n_{sh}} \theta (\bar{v}_{\xi} - \frac{n'_{sh}}{n_{sh}} \eta \bar{v}_{\eta} + \frac{v'_{sh}}{v_{sh}} \bar{v}) + \frac{v_{sh}}{u_{sh}} \theta \frac{\bar{\rho}\bar{v}}{n_{sh}} \bar{v}_{\eta}$$

$$- \frac{\kappa}{1+\kappa n_{sh}\eta} \frac{u_{sh}}{v_{sh}} \bar{\rho} \bar{u}^2 + \frac{p_{sh}}{\rho_{sh} u_{sh} v_{sh} n_{sh}} \bar{p}_{\eta} = 0 \qquad (D14)$$

where with full shock layer $\theta = 1$, and with the thin shock layer approximation $\theta = 0$. Using the equation of state, $\bar{p} = \bar{\rho}\bar{t}$, equation (D14) can be written in the following form

$$\frac{\bar{\rho}\bar{u}}{(1+\kappa n_{sh}\eta)} \rho_{sh} u_{sh} v_{sh} n_{sh} (\bar{v}_{\xi} - \frac{n'_{sh}}{n_{sh}} \eta \bar{v}_{\eta} + \frac{v'_{sh}}{v_{sh}} \bar{v})$$

$$+ \rho_{sh} v^2_{sh} \theta \bar{\rho} \bar{v} \bar{v}_{\eta} - \rho_{sh} u^2_{sh} n_{sh} \frac{\kappa}{1+\kappa n_{sh}\eta} \bar{\rho} \bar{u}^2$$

$$+ p_{sh} \bar{t} \bar{p}_{\eta} = 0 \qquad (D15)$$

The linearization of this equation gives,

$$c_1 \bar{u} + (a_2 \bar{v}_{\xi} + b_2 \bar{v}_{\eta} + c_2 \bar{v}) + (b_3 \bar{p}_{\eta} + c_3 \bar{p})$$

$$+ (c_4 \bar{t}) + d = 0 \qquad (D16)$$

The coefficients c_1, a_2, b_2,... etc., are given by the following relations,

$$c_1 = \gamma_2 \rho_{sh} u_{sh} \; [-2\kappa u_{sh} \; \bar{p}^o \; \bar{u}^o + \theta \; v_{sh} (\bar{v}^o_\xi - \frac{n'_{sh}}{n_{sh}} \eta \; \bar{v}^o_\eta + \frac{v'_{sh}}{v_{sh}} \bar{v}^o) \; \bar{p}^o]$$

$$a_2 = \gamma_2 \; \rho_{sh} v_{sh} u_{sh} \, \theta \, \bar{p}^o \; \bar{u}^o$$

$$b_2 = \rho_{sh} v_{sh} \, \theta \, \bar{p}^o \; (-\gamma_2 \frac{n'_{sh}}{n_{sh}} \eta \; u_{sh} \; \bar{u}^o + v_{sh} \; \bar{v}^o)$$

$$c_2 = \rho_{sh} v_{sh} \, \theta \, \bar{p}^o \; (\gamma_2 \frac{v'_{sh}}{v_{sh}} u_{sh} \; \bar{u}^o + v_{sh} \; \bar{v}^o_\eta)$$

$$b_3 = p_{sh} \; \bar{t}^o$$

$$c_3 = \rho_{sh} v_{sh} \; \{\gamma_2 [\theta \; u_{sh} (\bar{v}^o_\xi - \frac{n'_{sh}}{n_{sh}} \eta \; \bar{v}^o_\eta + \frac{v'_{sh}}{v_{sh}} \bar{v}^o) \; \bar{u}^o - \kappa \frac{u^2_{sh}}{v_{sh}} \; \bar{u}^{o2}] + \theta \, v_{sh} \; \bar{v}^o \; \bar{v}^o_\eta\}$$

$$c_4 = p_{sh} \; p^o_\eta$$

and

$$d_4 = - \, p_{sh} \; \bar{t}^o \; \bar{p}^o_\eta + 2 \; \rho_{sh} v_{sh} \; \{\frac{n_{sh}}{1+\kappa n_{sh}\eta} \, u_{sh} \; \bar{p}^o \; \bar{u}^o \; [\kappa \frac{u_{sh}}{v_{sh}} \bar{u}^o - \theta (\bar{v}^o_\xi - \frac{n'_{sh}}{n_{sh}} \eta \; \bar{v}^o_\eta + \frac{v'_{sh}}{v_{sh}} \bar{v}^o)] - \theta \; v_{sh} \; \bar{p}^o \; \bar{v}^o \; \bar{v}^o_\eta\} \qquad \text{(D17)}$$

where γ_2 is given by equation (D7).

Continuity Equation:

The continuity equation is given by equation (7) and after certain manipulations it can be written in the following form

$$\bar{\rho}(\alpha_1 \bar{u} + \alpha_2 \bar{v} + \alpha_3 \bar{u}_\xi + \alpha_4 \bar{v}_\eta + \alpha_5 \bar{u}_\eta) + \alpha_3 \bar{u} \bar{\rho}_\xi$$

$$+ (\alpha_4 \bar{v} + \alpha_5 \bar{u}) \bar{\rho}_\eta = 0 \qquad \text{(D18)}$$

Upon using the equation of state (9), equation (D18) can now be written as

$$\bar{p}\bar{t} (\alpha_1 \bar{u} + \alpha_2 \bar{v} + \alpha_3 \bar{u}_\xi + \alpha_4 \bar{v}_\eta + \alpha_5 \bar{u}_\eta)$$

$$+ \alpha_3 \bar{u}(\bar{t} \bar{p}_\xi - \bar{p} \bar{t}_\xi) + (\alpha_4 \bar{v} + \alpha_5 \bar{u})(\bar{t} \bar{p}_\eta - \bar{p} \bar{t}_\eta) = 0 \qquad \text{(D19)}$$

where the coefficients α_1, α_2,... etc., are given below,

$$\alpha_1 = \frac{n_{sh}}{1+\kappa n_{sh}\eta} \rho_{sh} u_{sh} \left[\frac{n_{sh_\xi} - n'_{sh}}{n_{sh}} \frac{u'_{sh}}{u_{sh}} + \frac{\rho'_{sh}}{\rho_{sh}} + \frac{(r+n_{sh}\eta\cos\phi)'}{(r+n_{sh}\eta\cos\phi)} - \frac{n'_{sh}\ \eta\cos\phi}{(r+n_{sh}\eta\cos\phi)}\right]$$

$$\alpha_2 = \rho_{sh} v_{sh} \left[\frac{n_{sh}\cos\phi}{(r+n_{sh}\eta\cos\phi)} + \frac{\kappa n_{sh}}{1+\kappa n_{sh}}\right]$$

$$\alpha_3 = \rho_{sh} u_{sh} \frac{n_{sh}}{1+\kappa n_{sh}\eta}$$

$$\alpha_4 = \rho_{sh} v_{sh}$$

and

$$\alpha_5 = -\rho_{sh} u_{sh} \frac{n'_{sh}}{n_{sh}} \eta \frac{n_{sh}}{1+\kappa n_{sh}\eta} \qquad \text{(D20)}$$

where the shock derivatives n_{sh_ξ} and n'_{sh} will result from the first and last terms of equation (8). In the present calculations, n_{sh_ξ} was evaluated as the shock thickness derivative at the time t^*, while n'_{sh} was evaluated at the time t^n. In the limit, when the solution convergence is achieved, n_{sh_ξ} value approaches n'_{sh} value.

The linearization of equation (D19), using equations (22a-c), gives the following equation,

$$(a_1\bar{u}_\xi + a_3\bar{p}_\xi + a_4\bar{t}_\xi) + (b_1\bar{u}_\eta + b_2\bar{v}_\eta + b_3\bar{p}_\eta + b_4\bar{t}_\eta)$$

$$+ (c_1\bar{u} + c_2\bar{v} + c_3\bar{p} + c_4\bar{t}) + d = 0 \qquad \text{(D21)}$$

where the coefficients of the linearized equation are given below,

$$a_1 = \gamma_2\ \rho_{sh}\ u_{sh}\ \bar{p}^o\ \bar{t}^o$$

$$a_3 = \gamma_2\ \rho_{sh}\ u_{sh}\ \bar{u}^o\ \bar{t}^o$$

$$a_4 = -\gamma_2\ \rho_{sh}\ u_{sh}\ \bar{u}^o\ \bar{p}^o$$

$$b_1 = -\gamma_2\ \frac{n'_{sh}}{n_{sh}}\ \rho_{sh}\ u_{sh}\ \eta\ \bar{p}^o\ \bar{t}^o$$

$$b_2 = \rho_{sh}\ v_{sh}\ \bar{p}^o\ \bar{t}^o$$

$$b_3 = (\rho_{sh}\ v_{sh}\ \bar{v}^o - \rho_{sh}\ u_{sh}\ \frac{n'_{sh}}{n_{sh}}\ \eta\ \gamma_2\ \bar{u}^o)\ \bar{t}^o$$

$$b_4 = -\ b_3\ p^o/\bar{t}^o$$

$$c_1 = \rho_{sh}\, u_{sh}\, \gamma_2 \{ [\frac{n_{sh_\xi} - n'_{sh}}{n_{sh}} + \frac{u'_{sh}}{u_{sh}} + \frac{\rho'_{sh}}{\rho_{sh}}$$

$$+ \gamma_3 (\frac{(r + n_{sh}\eta\cos\phi)'}{n_{sh}\cos\phi} - \frac{n'_{sh}}{n_{sh}}\eta)]\, \bar{p}^o\, \bar{t}^o$$

$$- \frac{n'_{sh}}{n_{sh}}\eta\, (\bar{t}^o\, \bar{p}^o_\eta - \bar{p}^o\, \bar{t}^o_\eta) + (\bar{t}^o\, \bar{p}^o_\xi - \bar{p}^o\, \bar{t}^o_\xi)\}$$

$$c_2 = \rho_{sh}\, v_{sh} \{ (\bar{t}^o\, \bar{p}^o_\eta - \bar{p}^o\, \bar{t}^o_\eta) + (\gamma_3 + \kappa\gamma_2)\, \bar{p}^o\, \bar{t}^o \}$$

$$c_3 = \rho_{sh}\, v_{sh}\, [\bar{t}^o\, \bar{v}^o_\eta + (\kappa\gamma_2 + \gamma_3)\, \bar{v}^o\, \bar{t}^o - \bar{v}^o\, \bar{t}^o_\eta]$$

$$+ \rho_{sh}\, v_{sh}\, \gamma_2\, \{\bar{t}^o\, (\bar{u}^o_\xi - \eta\frac{n'_{sh}}{n_{sh}}\bar{u}^o_\eta) - \bar{u}^o\, \bar{t}^o_\xi$$

$$+ \bar{t}^o\, \bar{u}^o\, [\frac{n_{sh_\xi} - n'_{sh}}{n_{sh}} + \frac{u'_{sh}}{u_{sh}} + \frac{\rho'_{sh}}{\rho_{sh}}$$

$$+ \gamma_3 (\frac{(r + n_{sh}\eta\cos\phi)'}{n_{sh}\cos\phi} - \frac{n'_{sh}}{n_{sh}}\eta)] - \frac{n'_{sh}}{n_{sh}}\eta\, \bar{u}^o\, \bar{p}^o_\eta\}$$

and

$$d = -\bar{2}\, \rho_{sh}\, v_{sh}\, [\bar{p}^o\, \bar{t}^o\, \bar{v}^o_\eta + (\kappa\gamma_2 + \gamma_3)\, \bar{v}^o\, \bar{p}^o\, \bar{t}^o + \bar{v}^o(\bar{t}^o\, \bar{p}^o_\eta - \bar{p}^o\, \bar{t}^o_\eta)]$$

$$-2\, \rho_{sh}\, u_{sh}\, \gamma_2\, \{\bar{p}^o\, \bar{t}^o(\bar{u}^o_\xi - \eta\frac{n'_{sh}}{n_{sh}}\bar{u}^o_\eta) + \bar{u}^o(\bar{t}^o\, \bar{p}^o_\xi - \bar{p}^o\, \bar{t}^o_\xi)$$

$$+ \bar{p}^o\, \bar{t}^o\, \bar{u}^o\, [\frac{n_{sh_\xi} - n'_{sh}}{n_{sh}} + \frac{u'_{sh}}{u_{sh}} + \frac{\rho'_{sh}}{\rho_{sh}} + \gamma_3 (\frac{(r + n_{sh}\eta\cos\phi)'}{n_{sh}\cos\phi} - \frac{n'_{sh}}{n_{sh}}\eta)]$$

$$- \frac{n'_{sh}}{n_{sh}}\eta\, \bar{u}^o\, (\bar{t}^o\, \bar{p}^o_\eta - \bar{p}^o\, \bar{t}^o_\eta)\}$$

APPENDIX E

FINITE DIFFERENCE EQUATIONS

This appendix gives the coefficients of the finite difference equations.

s-Momentum:

Using the difference quotients (27-29), equation (23) can be written in the following difference form,

$$(a_{11}u_{m,n-1} + a_{13}p_{m,n-1}) + (b_{11}u_{m,n} + b_{12}v_{m,n} + b_{13}p_{m,n} + b_{14}t_{m,n})$$

$$+ (c_{11}u_{m,n+1} + c_{13}p_{m,n+1}) = d_1$$

After dropping the subscript m, this equation becomes:

$$(a_{11}u_{n-1} + a_{13}p_{n-1}) + (b_{11}u_n + b_{12}v_n + b_{13}p_n + b_{14}t_n)$$

$$+ (c_{11}u_{n+1} + c_{13}p_{n+1}) = d_1 \tag{E1}$$

where

$$a_{11} = (2 - b_1\ \Delta\eta_n)/\Delta\eta_{n-1}\ (\Delta\eta_n + \Delta\eta_{n-1})$$

$$a_{13} = -\ b_3\ \Delta\eta_n/\Delta\eta_{n-1}\ (\Delta\eta_n + \Delta\eta_{n-1})$$

$$b_{11} = c_1 + [-2 + b_1(\Delta\eta_n - \Delta\eta_{n-1})]/\Delta\eta_n\Delta\eta_{n-1} + a_1/(\Delta\xi\ \text{CRNI})$$

$$b_{12} = c_2$$

$$b_{13} = a_3/(\Delta\xi\ \text{CRNI}) + c_3 + b_3\ (\Delta\eta_n - \Delta\eta_{n-1})/\Delta\eta_n\Delta\eta_{n-1}$$

$$b_{14} = c_4$$

$$c_{11} = (2 + b_1 \; \Delta\eta_{n-1})/\Delta\eta_n \; (\Delta\eta_n + \Delta\eta_{n-1})$$

$$c_{13} = (b_3 \; \Delta\eta_{n-1})/\Delta\eta_n \; (\Delta\eta_n + \Delta\eta_{n-1})$$

$$d_1 = R + (a_1 u_{m-1,n} + a_3 \; p_{m-1,n})/(\Delta\xi \; \mathrm{CRNI}) \tag{E2}$$

The coefficient CRNI is equal to 0.5 for Crank-Nicolson scheme, and 1 for pure implicit schemes. The coefficient R is given by,

$$R = -(d/\mathrm{CRNI}) - [(1-\mathrm{CRNI})/\mathrm{CRNI}][(u_{\eta\eta})_{m-1,n} + b_1 (u_\eta)_{m-1,n}$$
$$+ c_1 \; u_{m-1,n} + c_2 \; v_{m-1,n} + c_3 \; p_{m-1,n}$$
$$+ c_4 \; t_{m-1,n}$$

<u>Energy</u>:

Using the difference quotients (27-29), and dropping the subscript m, equation (24) can be written in the following finite difference form,

$$(a_{21} u_{n-1} + a_{23} p_{n-1} + a_{24} t_{n-1}) + (b_{21} u_n + b_{22} v_n + b_{23} p_n + b_{24} t_n)$$
$$+ (c_{21} u_{n+1} + c_{23} p_{n+1} + c_{24} t_{n+1}) = d_2 \tag{E3}$$

where

$$a_{21} = - b_1 \; \Delta\eta_n/\Delta\eta_{n-1} \; (\Delta\eta_n + \Delta\eta_{n-1})$$

$$a_{23} = - b_3 \; \Delta\eta_n/\Delta\eta_{n-1} \; (\Delta\eta_n + \Delta\eta_{n-1})$$

$$a_{24} = (2 - b_4 \, \Delta\eta_n)/\Delta\eta_{n-1} \, (\Delta\eta_n + \Delta\eta_{n-1})$$

$$b_{21} = c_1 + b_1 \, (\Delta\eta_n - \Delta\eta_{n-1})/\Delta\eta_n \, \Delta\eta_{n-1}$$

$$b_{22} = c_2$$

$$b_{23} = c_3 + a_3/\Delta\xi \text{ CRNI} + b_3 \, (\Delta\eta_n - \Delta\eta_{n-1})\Delta\eta_n\Delta\eta_{n-1}$$

$$b_{24} = c_4 + a_4/\Delta\xi \text{ CRNI} + b_4 \, (\Delta\eta_n - \Delta\eta_{n-1})/\Delta\eta_n\Delta\eta_{n-1} - 2/\Delta\eta_n\Delta\eta_{n-1}$$

$$d_2 = R + (a_3 p_{m-1,n} + a_4 t_{m-1,n})/\Delta\xi \text{ CRNI} \tag{E4}$$

and

$$R = - d/\text{CRNI} - [(1\text{-CRNI})/\text{CRNI}][(t_{\eta\eta})_{m-1,n} + b_1(u_\eta)_{m-1,n} + b_3(p_\eta)_{m-1,n} + b_4(t_\eta)_{m-1,n} + c_1 u_{m-1,n} + c_2 v_{m-1,n} + c_3 p_{m-1,n} + c_4 t_{m-1,n}]$$

<u>n-Momentum</u>:

Using the difference quotients (30, 31), evaluated at $(M,n+\frac{1}{2})$, and dropping the subscript m, equation (25) can be written in the following form,

$$(b_{31}u_n + b_{32}v_n + b_{33}p_n + b_{34}t_n) + (c_{31}u_{n+1} + c_{32}v_{n+1} + c_{33}p_{n+1} + c_{34}t_{n+1}) = d_3 \tag{E5}$$

where

$$b_{31} = \text{CRNI } \Delta\xi \, c_1$$

$$b_{32} = a_2 + \text{CRNI}\ \Delta\xi\ c_2 - 2\ \text{CRNI}\ b_2\ \Delta\xi/\Delta\eta$$

$$b_{33} = \text{CRNI}\ \Delta\xi\ c_3 - 2\ \text{CRNI}\ b_3\ \Delta\xi/\Delta\eta$$

$$b_{34} = \text{CRNI}\ \Delta\xi\ c_4$$

$$c_{31} = \text{CRNI}\ \Delta\xi\ c_1$$

$$c_{32} = a_2 + \text{CRNI}\ \Delta\xi\ c_2 + 2\ \text{CRNI}\ b_2\ \Delta\xi/\Delta\eta$$

$$c_{33} = \text{CRNI}\ \Delta\xi\ c_3 + 2\ \text{CRNI}\ b_3\ \Delta\xi/\Delta\eta$$

$$c_{34} = \text{CRNI}\ \Delta\xi\ c_4$$

$$\begin{aligned} d_3 = & - 2\ \Delta\xi\ d + [a_2 - (1 - \text{CRNI})\Delta\xi\ c_2][v_{m-1,n} + v_{m-1,n+1}] \\ & - (1 - \text{CRNI})\Delta\xi\ [c_1(u_{m-1,n} + u_{m-1,n+1}) \\ & + c_3(p_{m-1,n} + p_{m-1,n+1}) + c_4(t_{m-1,n} + t_{m-1,n+1})] \\ & - 2(1 - \text{CRNI})\Delta\xi\ [b_2(v_{m-1,n+1} - v_{m-1,n})/\Delta\eta \\ & + b_3(v_{m-1,n+1} - v_{m-1,n})/\Delta\eta] \end{aligned} \tag{E6}$$

Continuity:

Using the difference quotients (30, 31), and dropping the subscript m, equation (26) can be written in the following form,

$$\begin{aligned} & (a_{41}u_{n-1} + a_{42}v_{n-1} + a_{43}p_{n-1} + a_{44}t_{n-1}) \\ & + (b_{41}u_n + b_{42}v_n + b_{43}p_n + b_{44}t_n) = d_4 \end{aligned} \tag{E7}$$

where the coefficients a_{41}, a_{42}, ... etc., are given as,

$$a_{41} = a_1 + \text{CRNI}\ \Delta\xi\ c_1 - 2\ \text{CRNI}\ b_1\ \Delta\xi/\Delta\eta$$

$$a_{42} = \text{CRNI}\ \Delta\xi\ c_2 - 2\ \text{CRNI}\ b_2\ \Delta\xi/\Delta\eta$$

$$a_{43} = a_3 + \text{CRNI}\ \Delta\xi\ c_3 - 2\ \text{CRNI}\ b_3\ \Delta\xi/\Delta\eta$$

$$a_{44} = a_4 + \text{CRNI}\ \Delta\xi\ c_4 - 2\ \text{CRNI}\ b_4\ \Delta\xi/\Delta\eta$$

$$b_{41} = a_1 + \text{CRNI}\ \Delta\xi\ c_1 + 2\ \text{CRNI}\ b_1\ \Delta\xi/\Delta\eta$$

$$b_{42} = \text{CRNI}\ \Delta\xi\ c_2 + 2\ \text{CRNI}\ b_2\ \Delta\xi/\Delta\eta$$

$$b_{43} = a_3 + \text{CRNI}\ \Delta\xi\ c_3 + 2\ \text{CRNI}\ b_3\ \Delta\xi/\Delta\eta$$

$$b_{44} = a_4 + \text{CRNI}\ \Delta\xi\ c_4 + 2\ \text{CRNI}\ b_4\ \Delta\xi/\Delta\eta$$

$$\begin{aligned} d_4 = &- 2\ \Delta\xi\ d + [a_1 - c_1(1-\text{CRNI})\Delta\xi][u_{m-1,n+1} + u_{m-1,n}] \\ &- c_2(1-\text{CRNI})\Delta\xi\ (v_{m-1,n+1} + v_{m-1,n}) \\ &+ [a_3 - c_3(1-\text{CRNI})\Delta\xi]\ (p_{m-1,n+1} + p_{m-1,n}) \\ &+ [a_4 - c_4(1-\text{CRNI})\Delta\xi]\ (t_{m-1,n+1} + t_{m-1,n}) \\ &- 2(1-\text{CRNI})\ \frac{\Delta\xi}{\Delta\eta}\ [b_1(u_{m-1,n+1} - u_{m-1,n}) \\ &+ b_2(v_{m-1,n+1} - v_{m-1,n}) + b_3(p_{m-1,n+1} - p_{m-1,n}) \\ &+ b_4(t_{m-1,n+1} - t_{m-1,n})] \end{aligned} \qquad \text{(E8)}$$

APPENDIX F

SOLUTION OF THE DIFFERENCE EQUATIONS

The difference equations (32) to (35) are arranged into a system suitable to inversion with the recursion formulas (38a-d). The boundary conditions (36a-c) and the n-momentum equation (34) evaluated at n=1 are written together to obtain,

$$u_1 = 0$$

$$v_1 = 0$$

$$t_1 = t_w$$

$$(b_{31}u_1 + b_{32}v_1 + b_{33}p_1 + b_{34}t_1) + (c_{31}u_2 + c_{32}v_2 + c_{33}p_2 + c_{34}t_2) = d_3 \qquad \text{(F1)}$$

While the boundary conditions (37a), (37c) and (37d), and the continuity equation (35) evaluated at n=N are written together to obtain

$$u_N = 1$$

$$p_N = 1$$

$$t_N = 1$$

$$(a_{41}u_{N-1} + a_{42}v_{N-1} + a_{43}p_{N-1} + a_{44}t_{N-1}) + (b_{41}\, u_N + b_{42}\, v_N + b_3 p_N + b_{44}t_N) = d_4 \qquad \text{(F2)}$$

At the shock n=N, we still have an additional boundary condition, $v_N = 1$. The value of the shock thickness is iterated on to

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satisfy this boundary condition. The finite difference equations (32) to (35) evaluated at n = 2, 3,... and (N-1), are given by

$$(a_{11}u_{n-1} + a_{13}p_{n-1}) + (b_{11}u_n + b_{12}v_n + b_{13}p_n + b_{14}t_n)$$
$$+ (c_{11}u_{n+1} + c_{13}p_{n+1}) = d_1 \tag{F3}$$

$$(a_{21}u_{n-1} + a_{23}p_{n-1} + a_{24}t_{n-1}) + (b_{21}u_n + b_{22}v_n + b_{23}p_n + b_{24}t_n)$$
$$+ (c_{21}u_{n+1} + c_{23}p_{n+1} + c_{24}t_{n+1}) = d_2 \tag{F4}$$

$$(b_{31}u_n + b_{32}v_n + b_{33}p_n + b_{34}t_n)$$
$$+ (c_{31}u_{n+1} + c_{32}v_{n+1} + c_{33}p_{n+1} + c_{34}t_{n+1}) = d_3 \tag{F5}$$

$$(a_{41}u_{n-1} + a_{42}v_{n-1} + a_{43}p_{n-1} + a_{44}t_{n-1})$$
$$+ (b_{41}u_n + b_{42}v_n + b_{43}p_n + b_{44}t_n) = d_4 \tag{F6}$$

Equations (F1) to (F6) in this arranged form, represent a system of block tridiagonal equations. Their solution is obtained with the following recursion relations,

$$u_{n+1} = D_{u_{n+1}}u_n + E_{u_{n+1}}v_n + F_{u_{n+1}}p_n + G_{u_{n+1}}t_n + H_{u_{n+1}}$$

$$v_{n+1} = D_{v_{n+1}}u_n + E_{v_{n+1}}v_n + F_{v_{n+1}}p_n + G_{v_{n+1}}t_n + H_{v_{n+1}}$$

$$p_{n+1} = D_{p_{n+1}}u_n + E_{p_{n+1}}v_n + F_{p_{n+1}}p_n + G_{p_{n+1}}t_n + H_{p_{n+1}}$$

$$t_{n+1} = D_{t_{n+1}}u_n + E_{t_{n+1}}v_n + F_{t_{n+1}}p_n + G_{t_{n+1}}t_n + H_{t_{n+1}} \tag{F7}$$

The quantities D, E, F, G and H are obtained at all the grid points by the following procedure. By substituting (F7) into (F3), (F4) and (F5), to eliminate variables with subscripts n+1, the following equations are obtained,

$$\bar{b}_{11}u_n + \bar{b}_{12}v_n + \bar{b}_{13}p_n + \bar{b}_{14}t_n = \bar{d}_1 - a_{11}u_{n-1} - a_{13}p_{n-1} \quad \text{(F8)}$$

$$\bar{b}_{21}u_n + \bar{b}_{22}v_n + \bar{b}_{23}p_n + \bar{b}_{24}t_n = \bar{d}_2 - a_{21}u_{n-1} - a_{23}p_{n-1} - a_{24}t_{n-1} \quad \text{(F9)}$$

and

$$\bar{b}_{31}u_n + \bar{b}_{32}v_n + \bar{b}_{33}p_n + \bar{b}_{34}t_n = \bar{d}_3 \quad \text{(F10)}$$

where

$$\bar{b}_{11} = b_{11} + c_{11}D_{u_{n+1}} + c_{13}D_{p_{n+1}}$$

$$\bar{b}_{12} = b_{12} + c_{11}E_{u_{n+1}} + c_{13}E_{p_{n+1}}$$

$$\bar{b}_{13} = b_{13} + c_{11}F_{u_{n+1}} + c_{13}F_{p_{n+1}}$$

$$\bar{b}_{14} = b_{14} + c_{11}G_{u_{n+1}} + c_{13}G_{p_{n+1}}$$

$$\bar{d}_1 = d_1 - c_{11}H_{u_{n+1}} - c_{13}H_{p_{n+1}}$$

$$\bar{b}_{21} = b_{21} + c_{21}D_{u_{n+1}} + c_{23}D_{p_{n+1}} + C_{24}D_{t_{n+1}}$$

$$\bar{b}_{22} = b_{22} + c_{21}E_{u_{n+1}} + c_{23}E_{p_{n+1}} + c_{24}E_{t_{n+1}}$$

$$\bar{b}_{23} = b_{23} + c_{21}F_{u_{n+1}} + c_{23}F_{p_{n+1}} + c_{24}F_{t_{n+1}}$$

$$\bar{b}_{24} = b_{24} + c_{21}G_{u_{n+1}} + c_{23}G_{p_{n+1}} + c_{24}G_{t_{n+1}}$$

$$\bar{d}_2 = d_2 - c_{21}H_{u_{n+1}} - c_{23}H_{p_{n+1}} - c_{24}H_{t_{n+1}}$$

$$\bar{b}_{31} = b_{31} + c_{31}D_{u_{n+1}} + c_{32}D_{v_{n+1}} + c_{33}D_{p_{n+1}} + c_{34}D_{t_{n+1}}$$

$$\bar{b}_{32} = b_{32} + c_{31}E_{u_{n+1}} + c_{32}E_{v_{n+1}} + c_{33}E_{p_{n+1}} + c_{34}E_{t_{n+1}}$$

$$\bar{b}_{33} = b_{33} + c_{31}F_{u_{n+1}} + c_{32}F_{v_{n+1}} + c_{33}F_{p_{n+1}} + c_{34}F_{t_{n+1}}$$

$$\bar{b}_{34} = b_{34} + c_{31}G_{u_{n+1}} + c_{32}G_{v_{n+1}} + c_{33}G_{p_{n+1}} + c_{34}G_{t_{n+1}}$$

$$\bar{d}_3 = d_3 - c_{31}H_{u_{n+1}} - c_{32}H_{v_{n+1}} - c_{33}H_{p_{n+1}} - c_{34}H_{t_{n+1}} \tag{F11}$$

By solving equations (F6), (F8), (F9) and (F10), the variables u_n, v_n, p_n and t_n can be obtained in terms of u_{n-1}, v_{n-1}, p_{n-1} and t_{n-1}, and in the following form

$$u_n = D_{u_n}u_{n-1} + E_{u_n}v_{n-1} + F_{u_n}p_{n-1} + G_{u_n}t_{n-1}$$

$$v_n = D_{v_n}u_{n-1} + E_{v_n}v_{n-1} + F_{v_n}p_{n-1} + G_{v_n}t_{n-1}$$

$$p_n = D_{p_n}u_{n-1} + E_{p_n}v_{n-1} + F_{p_n}p_{n-1} + G_{p_n}t_{n-1}$$

$$t_n = D_{t_n}u_{n-1} + E_{t_n}v_{n-1} + F_{t_n}p_{n-1} + G_{t_n}t_{n-1} \tag{F12}$$

where the coefficients D_{u_n}, E_{u_n},... etc., are given by,

$$D_{u_n} = (-\, a_{11}\bar{D}_{11} + a_{21}\bar{D}_{21} + a_{41}\bar{D}_{41})/\mathrm{Det}$$

$$E_{u_n} = (a_{42}\bar{D}_{41})/\mathrm{Det}$$

$$F_{u_n} = (-\, a_{13}\bar{D}_{11} + a_{23}\bar{D}_{21} + a_{43}\bar{D}_{41})/\mathrm{Det}$$

$$G_{u_n} = (-\, a_{14}\bar{D}_{11} + a_{24}\bar{D}_{21} + a_{44}\bar{D}_{41})/\mathrm{Det}$$

$$H_{u_n} = (\bar{d}_1\bar{D}_{11} - \bar{d}_2\bar{D}_{21} + \bar{d}_3\bar{D}_{31} - d_4\bar{D}_{41})/\mathrm{Det}$$

$$D_{v_n} = (a_{11}\bar{D}_{12} - a_{21}\bar{D}_{22} - a_{41}\bar{D}_{42})/\mathrm{Det}$$

$$E_{v_n} = (-\, a_{41}\bar{D}_{42})/\mathrm{Det}$$

$$F_{v_n} = (a_{13}\bar{D}_{12} - a_{23}\bar{D}_{22} - a_{43}\bar{D}_{42})/\mathrm{Det}$$

$$G_{v_n} = (a_{14}\bar{D}_{12} - a_{24}\bar{D}_{22} - a_{44}\bar{D}_{42})/\mathrm{Det}$$

$$H_{v_n} = (-\, \bar{d}_1\bar{D}_{12} + \bar{d}_2\bar{D}_{22} - \bar{d}_3\bar{D}_{32} + d_4\bar{D}_{42})/\mathrm{Det}$$

$$D_{p_n} = (-\, a_{11}\bar{D}_{13} + a_{21}\bar{D}_{23} + a_{41}\bar{D}_{43})/\mathrm{Det}$$

$$E_{p_n} = (a_{42}\bar{D}_{43})/\mathrm{Det}$$

$$F_{p_n} = (-\, a_{13}\bar{D}_{13} + a_{23}\bar{D}_{23} + a_{43}\bar{D}_{43})/\mathrm{Det}$$

$$G_{p_n} = (-\,a_{14}\bar{D}_{13} + a_{24}\bar{D}_{23} + a_{44}\bar{D}_{43})/\mathrm{Det}$$

$$H_{p_n} = (\bar{d}_1\bar{D}_{13} - \bar{d}_2\bar{D}_{23} + \bar{d}_3\bar{D}_{33} - d_4\bar{D}_{43})/\mathrm{Det}$$

$$D_{t_n} = (a_{11}\bar{D}_{14} - a_{21}\bar{D}_{24} - a_{41}\bar{D}_{44})/\mathrm{Det}$$

$$E_{t_n} = (-\,a_{42}\bar{D}_{44})/\mathrm{Det}$$

$$F_{t_n} = (a_{13}\bar{D}_{14} - a_{23}\bar{D}_{24} - a_{43}\bar{D}_{44})/\mathrm{Det}$$

$$G_{t_n} = (-\,a_{14}\bar{D}_{14} - a_{24}\bar{D}_{24} - a_{44}\bar{D}_{44})/\mathrm{Det}$$

$$H_{t_n} = (-\,\bar{d}_1\bar{D}_{14} + \bar{d}_2\bar{D}_{24} - \bar{d}_3\bar{D}_{34} + d_4\bar{D}_{44})/\mathrm{Det} \qquad \text{(F13)}$$

and the coefficients $\bar{D}_{11}$, $\bar{D}_{21}$,... etc., are given by the following determinants,

$$\bar{D}_{11} = \begin{vmatrix} \bar{b}_{22} & \bar{b}_{23} & \bar{b}_{24} \\ \bar{b}_{32} & \bar{b}_{33} & \bar{b}_{34} \\ b_{42} & b_{43} & b_{44} \end{vmatrix}, \quad \bar{D}_{21} = \begin{vmatrix} \bar{b}_{12} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{32} & \bar{b}_{33} & \bar{b}_{34} \\ b_{42} & b_{43} & b_{44} \end{vmatrix}$$

$$\bar{D}_{31} = \begin{vmatrix} \bar{b}_{12} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{22} & \bar{b}_{23} & \bar{b}_{24} \\ b_{42} & b_{43} & b_{44} \end{vmatrix}, \quad D_{41} = \begin{vmatrix} \bar{b}_{12} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{22} & \bar{b}_{23} & \bar{b}_{24} \\ \bar{b}_{32} & \bar{b}_{33} & \bar{b}_{34} \end{vmatrix}$$

$$\bar{D}_{12} = \begin{vmatrix} \bar{b}_{21} & \bar{b}_{23} & \bar{b}_{24} \\ \bar{b}_{31} & \bar{b}_{33} & \bar{b}_{34} \\ b_{41} & b_{43} & b_{44} \end{vmatrix} , \quad D_{22} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{31} & \bar{b}_{33} & \bar{b}_{34} \\ b_{41} & b_{43} & b_{44} \end{vmatrix}$$

$$\bar{D}_{32} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{21} & \bar{b}_{23} & \bar{b}_{24} \\ b_{41} & b_{43} & b_{44} \end{vmatrix} , \quad \bar{D}_{42} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{21} & \bar{b}_{23} & \bar{b}_{24} \\ \bar{b}_{31} & \bar{b}_{33} & \bar{b}_{34} \end{vmatrix}$$

$$\bar{D}_{13} = \begin{vmatrix} \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{24} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{34} \\ b_{41} & b_{42} & b_{44} \end{vmatrix} , \quad \bar{D}_{23} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{14} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{34} \\ b_{41} & b_{42} & b_{44} \end{vmatrix}$$

$$\bar{D}_{33} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{14} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{24} \\ b_{41} & b_{42} & b_{44} \end{vmatrix} , \quad \bar{D}_{43} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{14} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{24} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{34} \end{vmatrix}$$

$$\bar{D}_{14} = \begin{vmatrix} \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \\ b_{41} & b_{42} & b_{43} \end{vmatrix} , \quad \bar{D}_{24} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \\ b_{41} & b_{42} & b_{43} \end{vmatrix}$$

$$\bar{D}_{34} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} \\ b_{41} & b_{42} & b_{43} \end{vmatrix} , \quad \bar{D}_{44} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \end{vmatrix}$$

and

$$\text{Det} = \begin{vmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} & \bar{b}_{14} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} & \bar{b}_{24} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} & \bar{b}_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{vmatrix}$$

Equation (E13) gives the necessary relations for the coefficients D_{u_n}, D_{v_n}, D_{p_n},... etc. Given the coefficients at n=N, the quantities at all grid points can be calculated with a sweep from grid point n=N-1 to grid point n=2. The values at n=N are evaluated using equation (E2) which gives,

$$D_{u_N} = E_{u_N} = F_{u_N} = G_{u_N} = 0 \ , \quad H_{u_N} = 1$$

$$D_{p_N} = E_{p_N} = F_{p_N} = G_{p_N} = 0 \ , \quad H_{p_N} = 1$$

$$D_{t_N} = E_{t_N} = F_{t_N} = G_{t_N} = 0 \ , \quad H_{t_N} = 1$$

$$D_{v_N} = - a_{41}/b_{42} \ , \quad E_{v_N} = - a_{42}/b_{42}$$

$$F_{v_N} = - a_{43}/b_{42} \ , \quad G_{v_N} = - a_{44}/b_{42}$$

and

$$H_{v_N} = (d_4 - b_{41} u_N - b_{43} p_N - b_{44} t_N)/b_{42} \tag{F14}$$

Once all the coefficients of the recursion formulas are obtained, the solution of the difference equations (32) to (35) are obtained with a backward sweep from n=1 to n=N-2 using

equations (E7), provided that u_1, v_1, t_1 and p_1 are known from (E1) and given by,

$$u_1 = 0$$

$$v_1 = 0$$

$$t_1 = t_w$$

and

$$p_1 = [(d_3 - c_{31} H_{u_2} - c_{32} H_{v_2} - c_{33} H_{p_2} - c_{34} H_{t_2}) - (b_{34} + c_{31} G_{u_2} + c_{32} G_{v_2} + c_{33} G_{p_2} + c_{34} G_{t_2}) t_w] / (b_{33} + c_{31} F_{u_2} + c_{32} F_{v_2} + c_{33} F_{p_2} + c_{34} F_{t_2}) \qquad \text{(F15)}$$

In Appendix G, the subroutine DEQSO of the computer program provides the solution of the present finite difference equations.

APPENDIX G

COMPUTER PROGRAM

The following computer program, written in Fortran IV, was used to solve the full viscous shock layer equations as a coupled set. The input quantities are:

Main Program

RMAC	Free stream Mach number, M_∞.
BO	Wall to stagnation temperature ratio, t_w/t_o.
REYIN	Free stream Reynolds number, Re_∞.
TEMP	Free stream temperature, T_∞ in °R.
ITHIN	0 for thin layer, 1 for full layer.
IE	Number of mesh points in the η-direction.
IEND	Number of mesh points in the s-direction.
IPRINT	Number of s steps between successive printings, $s \leq 1.0$.
IPRIN1	Number of s steps between successive printings, $s \geq 1.0$.
DS	s step size.
DT	Time step size.
GAM	Ratio of specific heats, γ.
SIGM	Prandtl number, σ.
XFACT	Convergence criteria for local iterations.

Block Data

XNSH	Initial shock thickness, assumed constant over the body (0.1072).

Output Quantities:

MINF	Free stream Mach number.
TINF	Free stream temperature.
TW/TO	Wall to stagnation temperature ratio, t_w/t_o.
PR	Prandtl number, σ.
S	ξ, surface distance.
X	Axial distance measured from the body nose.
R	Body radius measured from axis.
NSH	Shock stand off distance normal to body surface.
XSH	Shock axial distance measured from body nose.
RSH	Shock radius measured from axis.
EPS	Defined as, $[\mu^* (U_\infty^{*2}/C_p^*)/\rho_\infty^* U_\infty^* a^*]^{1/2}$
NO ITER	Number of local iterations.
NTIME	Number of time iteration cycles.
CF	Skin friction coefficient, $2\tau_w^*/\rho_\infty^* U_\infty^{*2}$.
HEAT	Wall heat transfer, $q_w^*/\rho_\infty^* U_\infty^{*3}$.
STAN	Stanton number, $q_w/(H_O-H_N)$.
PWALL	Pressure at the wall, $p^*/\rho_\infty^* U_\infty^{*2}$.
PW/PO	Pressure at the wall/Nose stagnation pressure.
UUS	u-component of velocity behind the shock.
VVS	v-component of velocity behind the shock.
PPS	Pressure behind the shock.
TTS	Temperature behind the shock.
RRS	Density behind the shock.
N/NSH	η coordinate.
U/USH	$\bar{u}$; normalized tangential velocity.

V	Velocity component normal to the body surface.
P/PSH	Normalized pressure.
T/TSH	Normalized temperature.
R/RSH	Normalized density.
MACH	Mach number.
PITO	Pitot pressure.

List of Subroutines

SHVALS	Calculates properties behind the shock.
GEOM	Calculates body geometry for any given longitudinal location, s. This subroutine is only for hyperboloid. The asymptotic angle is 45°. It can be changed by correcting the second statement (ANG = ...).
DEQSO	Solves coupled set of equations simultaneously.
DERIV	Calculates the derivatives for any shock shape.
MANISH	Calculates the new shock shape, final sweep.
BLOCK DATA	Ininital shock shape.

```
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/DEQS1/U1(111),V1(111),P1(111),T1(111),UC(111),VC(111),
     1   PC(111),TC(111),U1N(111),V1N(111),P1N(111),T1N(111),UCN(111),
     2          VCN(111),PCN(111),TCN(111),U1NN(111),T1NN(111),
     3          R1(111),R2(111),RC(111)
      DIMENSION VISC(111),RVISC(111),RNSH(111),RCSF(111),RCSFP(111),
     1          PFAC(111),XM(111),PITO(111)
      DIMENSION US(111),VS(111),PS(111),TS(111)
      DIMENSION PO(111),PON(111),PA(111),PAN(111)
      DIMENSION P21(111),P21N(111),P33(111),P33N(111)
      DIMENSION VO(111),VON(111),VG(111),VGN(111),VGS(111),VCI(111, 2 )
      DIMENSION  VVSG( 2 ),VSPP( 2 )
      COMMON/DEQSS/ U2(111),V2(111),P2(111),T2(111),U2N(111),V2N(111),
     1          P2N(111),T2N(111),U2NN(111),T2NN(111)
      COMMON/DEQS/A11(111),A13(111),B11(111),B12(111),B13(111),B14(111),
     1   C11(111),C13(111), D1(111),A21(111),A23(111),A24(111),B21(111),
     2   B22(111),B23(111),B24(111),C21(111),C23(111),C24(111), D2(111),
     3   B31(111),B32(111),B33(111),B34(111),C31(111),C32(111),C33(111),
     4   C34(111), D3(111),A41(111),A42(111),A43(111),A44(111),B41(111),
     5   B42(111),B43(111),B44(111), D4(111)
      COMMON/MAINN/CRNI,DS,DN(111),XN(111),IM,IE
      COMMON/SHCKG/XNSH(202),XNSP(202),XNSPP(202),YNSH(202),YNSP(202),
     1               YNSPP(202),AD(202),AB(202),AC(202)
      COMMON /INSH/ CONO   ,   GAM    ,   S      ,   UPSH   ,   XNS    ,
     1              EPS    ,   RMAC   ,   TPSH   ,   VISCO
      COMMON/OUTSH/ PPS    ,   RRS    ,   TTS    ,   UUS1   ,   VVS    ,
     1              PPS1   ,   RRS1   ,   TTS1   ,   UUS2   ,   VVS1   ,
     2              PSP    ,   RRS2   ,   TSP    ,   USP    ,   VVS2   ,
     3              PPS2   ,   RSP    ,   TTS2   ,   UUS    ,   VSP
      DIMENSION YYYY(202), XXXX(202)
      DATA BFULL/'FULL-'/,BTHIN/'THIN-'/
      READ(5,1001) RMAC,BO,REYIN,TEMP
      READ(5,1002) ITHIN,IE,IEND,IPRINT,IPRIN1
      READ(5,1003) DS,DT,GAM,SIGM,XFACT
 1001 FORMAT(4F10.3)
 1002 FORMAT(5I4)
 1003 FORMAT(4F7.3,D10.3)
      CRNI=1.0D0
      THIN=ITHIN
      WRITE(6,900)
  900 FORMAT(1H1,45X,'SHOCK LAYER PROGRAM (W. HOSNY)'//3X,'INPUT DATA'/)
      WRITE(6,901) RMAC,TEMP,REYIN,BO,GAM,SIGM
  901 FORMAT(6X,'MINF',6X,'TINF' ,6X,'REYIN',7X,'TW/TO',5X,'GAM',7X,
     1        ' PR ',/5X,F6.2,4X,F6.2,4X,F7.2,6X,F4.2,6X,F3.1,6X,F4.2)
      CNT=BTHIN
      IF(ITHIN.EQ.1) CNT=BFULL
      WRITE(6,902) CNT,IE,IEND,DS,DT
  902 FORMAT(/3XA5,'SHOCK LAYER',5X,'NO. OF STEPS IN N =',I4,4X,
     1        'NO. OF STEPS IN S =',I4,4X,'S STEP SIZE =',F5.3,4X,
```

```
  2          'T STEP SIZE =',F6.1)
     DO 10 I=1,IEND
 10  XNSH(I)=0.0D0
     IM=IE-1
     YIM=IM
     DY=1.D0/YIM
     XN(1)=0.0D0
     DO 15 N=1,IM
     DN(N)=DY
 15  XN(N+1)=XN(N)+DN(N)
     YNSH(1)=0.0
     CALL DERIV(DS,IEND,1)
     IEND1=IEND+1
     WRITE(6,903) ( XNSH(I),I=1,IEND1)
     WRITE(6,904) ( YNSH(I),I=1,IEND1)
903   FORMAT(/3X,'INITIAL SHOCK SHAPE',//5X,'SHOCK THICKNESS'/
  1          30(3X,10F10.4/))
904  FORMAT(//5X,'SHOCK RADUIS'/30(3X,10F10.4/))
     NTIME=0
     NTIME1=0
     TIME=0.0
 20  NTIME=NTIME+1
     TIME=TIME+DT
     RSH=0.0
     UPSH=0.0
     TPSH=0.0
     VISCD=0.0D0
     CONO=0.0D0
     XNS=XNSH(1)
     XNS1=XNS
     CNS=(XNS1+XNS)/2.
     DS2=DS/2.0D0
     CK=1.0D0
     CSF=0.0D0
     SIF=1.0D0
     PHIC=DARCOS(0.0D0)
     RS=0.0D0
     RS2=0.0D0
     XB=0.0
     CDF=0.0
     CDP=0.0
     CDP1=0.0
     CDP2=0.0
     CDF1=0.0
     CDF2=0.0
     CDPD=0.0
     CDFD=0.0
     POTP=((GAM+1.0)*RMAC*RMAC/2.0)**(GAM/(GAM-1.0))/(GAM*RMAC*RMAC*
  1       (2.0*GAM*RMAC*RMAC/(GAM+1.0)-(GAM-1.0)/(GAM+1.0))**
  2       (1.0/(GAM-1.0)))
```

```
      TW=BO*(1.0/((GAM-1.0)*RMAC*RMAC)+0.50)
      TB=TW*((GAM-1.0)*RMAC*RMAC*TEMP)
      CONP=198.6/((GAM-1.)*RMAC*RMAC*TEMP)
      VISRA=(1.0+CONP)*(1.0/((GAM-1.)*RMAC*RMAC))**1.5/(1./((GAM-1.)*
     1        RMAC*RMAC)+CONP)
      EPS=1.0/DSQRT(VISRA*REYIN)
      CALL SHVALS(1.0D0,0.0D0,1.0D0,0.0D0,TTSO,VVSO,UUSO,PPSO,1)
      TTS=TTSO
      DO 100 N=1,IE
      RNSH(N)=CNS/(1.+CK*CNS*XN(N))
      RCSF(N)=CNS/(1.+CK*CNS*XN(N))
      RCSFP(N)=1.0+CNS*XN(N)
      U1(N)=XN(N)
      U2(N)=XN(N)
      U1N(N)=1.0
      U2N(N)=1.0
      U1NN(N)=0.0
      UC(N)=XN(N)
      UCN(N)=1.0
      US(N)=0.0
      VS(N)=0.0
      PS(N)=0.0
      TS(N)=0.0
      V1(N)=XN(N)
      V2(N)=XN(N)
      VC(N)=XN(N)
      T1(N)=1.0-(1.0-XN(N))*(1.0-TW/TTSO)
      T2(N)=T1(N)
      T1N(N)=1.0-TW/TTSO
      T2N(N)=T1N(N)
      T1NN(N)=0.0
      TC(N)=T1(N)
      TCN(N)=T1N(N)
      VISC(N)=(TTS+CONP)*TC(N)**1.5/(TTS*TC(N)+CONP)
      RVISC(N)=(TTS*TC(N)+3.0*CONP)/(2.0*TC(N)*(TTS*TC(N)+CONP))*TCN(N)
      P1(N)=1.0
      P2(N)=1.0
      PC(N)=1.0
      PA(N)=1.0
      PO(N)=1.0
      PON(N)=0.0
      RC(N)=PC(N)/TC(N)
      PCN(N)=0.0
      P1N(N)=0.0
      PAN(N)=0.0
      P33(N)=0.0D0
      P21(N)=1.0D0
  100 P2N(N)=0.0
      VISCO=(1.0+CONP)*TTS**1.5/(TTS+CONP)
      CONO=VISCO/SIGM
```

```
      DO 5000 I=1,IEND
      YI=I
      S=(YI-1.0D0)*DS
      CALL GEOM(S,DS2,RS2,CK2,CSF2,SIF2,XB2)
      IF(I.EQ.1) CK1=CK2
      CKP=(CK2-CK1)/DS
      PHIP=-CK
      PHI = DARCOS(CSF2)
      XNSPM=(XNSP(I)+XNSP(I+1))/2.0D0
      CNSN=XNSH(I)
      XNSN=(XNSH(I)+XNSH(I+1))/2.0
      IF(I.EQ.1) CNSN=XNSN
      ALP = PHI + DATAN(XNSPM/(1.0+CK2*XNSN))
      ALPC=PHIC+DATAN(XNSP(I)/(1.0+CK*CNSN))
      SP = DSIN(ALP)
      CP = DCOS(ALP)
      SPB=SP*SIF2+CP*CSF2
      CPB=CP*SIF2-SP*CSF2
      IF(S.LT.0.0001) GO TO 120
      RSCP=(RS2-RS1)/DS
      CSFP=(CSF2-CSF1)/DS
  120 CONTINUE
      NITT=0
 2000 NITT=NITT+1
      CALL SHVALS ( SP, CP, SPB, CPB, TTSH, VRSH, URSH, PPSH, 2)
      USD=USP
      RSD=RSP
C
      GAMQ=GAM*GAM
      GAMM=GAM-1.0
      GAMB=GAM+1.0
      GAMT=GAM*GAMB/GAMM-2.0*GAM*GAMM/GAMB
      CPHC=DCOS(PHIC)
      SPC=DSIN(ALPC)
      CPC=DCOS(ALPC)
      S2PC=2.0*SPC*CPC
      SPCQ=SPC*SPC
      SPCT=SPCQ*SPC
      S2PPH=DSIN(2.0*ALPC-PHIC)
      SPPH=DSIN(ALPC-PHIC)
      CPPH=DCOS(ALPC-PHIC)
      C2PPH=DCOS(2.0*ALPC-PHIC)
      RMACQ=RMAC*RMAC
      AR=2.0*GAM*RMACQ*SPCQ-GAMM
      AK1P=2.0*GAMQ*RMACQ*S2PC/(AR*GAMB)
     1      +4.0*GAM*CPC/(GAMB*RMACQ*AR*SPCT)
     2      -4.0*GAM*GAMQ*RMACQ*RMACQ*S2PC*SPCQ/(AR*AR*GAMB)
     3      -2.0*GAM*RMACQ*S2PC*GAMT/(AR*AR)
     4      +4.0*GAMQ*S2PC/(GAMB*AR*AR*SPCQ)
      AK1=-(1.0-(GAMM/2.0)*(TTS/PPS))*S2PPH+SPC*SPPH*GAMM*AK1P/GAM
```

```
      AK2=CPC*SPPH-(GAMM/GAM)*(TTS/PPS)*SPC*CPPH
      AK3=2.0*S2PC/GAMB
      AK4=2.0*GAM*S2PC/(GAMB*GAMB)+4.0*CPC/(GAMB*GAMB*RMACQ*RMACQ*SPCT)
      AK5=(1.0-(GAMM/GAM)*(TTS/PPS))*C2PPH-SPC*CPPH*AK1P*GAMM/GAM
      AK6=-CPC*CPPH-SPC*SPPH*(GAMM/GAM)*(TTS/PPS)
      ALPP=(XNSPP(1)-(CNS-CNSN)*2.0/(3.0*DT))/(1.0+CNS )-1.0
      VSP=0.0
      PSP=0.0
      TSP=0.0
      RSP=0.0
      IF(I.EQ.1) GO TO 123
      ALPP= (CPPH*CPPH*(YNSPP(I)-(RSH-YNSH(I))*2.0/DT)-YNSP(I)*(CK*2.0*
     1     SPPH*CPPH+CNS *CKP*CPPH*CPPH/(1.0+CK*CNS )))/((1.0+CK*CNS )*
     2     CPHC)
      VSP=AK5*ALPP+AK6*PHIP
      PSP=AK3*ALPP
      TSP=AK4*ALPP
      RSP=(GAM/GAMM)*(PSP*TTS-TSP*PPS)/(TTS*TTS)
  123 CONTINUE
      USP=AK1*ALPP+AK2*PHIP
C
      IF(I.EQ.1) VVM=VVS
      IF(I.EQ.1) VSD=VSP
      IF(I.NE.1) VVM=1.0D0
      IF(I.NE.1) VSD=0.0D0
      DO 110 N=1,IE
      IF(S.GE.0.0001) GO TO 108
      PFAC(N)=4.0D0*(PA(N)+(PPS2/PPS0-2.0D0)*PO(N))/(UUS2*DS)
     1          -XNSP(2)*XN(N)*PON(N)/(2.0D0*UUS2*CNS)
      GO TO 109
  108 PFAC(N)=(PS(N)-XNSP(I)*XN(N)*PCN(N)/CNS+PSP*PC(N)/PPS)/UUS
  109 CONTINUE
      RNSH(N)=CNS/(1.+CK*CNS*XN(N))
      VISC(N)=(TTS+CONP)*TC(N)**1.5/(TTS*TC(N)+CONP)
      RVISC(N)=(TTS*TC(N)+3.0*CONP)/(2.0*TC(N)*(TTS*TC(N)+CONP))*TCN(N)
  110 CONTINUE
      VISCO=(1.0+CONP)*TTS**1.5/(TTS+CONP)
      CONO=VISCO/SIGM
      REFAC=RRS*VVM*CNS/(EPS*EPS*VISCO)
      IF(S.LT.0.0001) GO TO 170
      XNSPT=(XNS-XNS1)/DS
      DO 165 N=1,IE
      RCSF(N)=CSF*CNS/(RS+CNS*XN(N)*CSF)
      RCSFP(N)=RSCP+XN(N)*(XNSPT*CSF+CNS*CSFP)
  165 CONTINUE
  170 CONTINUE
C   S MOMENTUM COEFFICIENTS
      DO 400 N=2,IM
      A1=-REFAC*RNSH(N)*UUS*RC(N)*UC(N)/(VISC(N)*VVM)
      B1=REFAC*(RNSH(N)*UUS*XNSP(I)*XN(N)*RC(N)*UC(N)/(VVM*CNS*VISC(N))
```

```
     1   -RC(N)*VC(N)/VISC(N))+RVISC(N)+CK*RNSH(N)+RCSF(N)
      C1=-RNSH(N)*(REFAC*RC(N)*(2.0*USP*UC(N)/(VVM*VISC(N))+CK*VC(N)
     1   /VISC(N)-UUS*XNSP(I)*XN(N)*UCN(N)/(VVM*CNS*VISC(N))+UUS*US(N)/
     2   (VVM*VISC(N)))+CK*(RVISC(N)+CK*RNSH(N)+RCSF(N)))
      C2=-REFAC*RC(N)*(UCN(N)/VISC(N)+CK*RNSH(N)*UC(N)/VISC(N))
      C4=-REFAC*RC(N)*(RNSH(N)*( UUS*XNSP(I)*XN(N)*UCN(N)/(VVM*CNS*
     1   VISC(N))-USP*UC(N)/(VVM*VISC(N))-UUS*US(N)/(VVM*VISC(N))-CK*
     2   VC(N)/VISC(N))*UC(N)    -VC(N)*UCN(N)/VISC(N))/TC(N)
      D=REFAC*(RNSH(N)*((-UUS*XNSP(I)*XN(N)*UCN(N)/(VVM*CNS)+USP*UC(N)
     1  /VVM+UUS*US(N)/VVM)*RC(N)*UC(N)/VISC(N)+CK*RC(N)*UC(N)*VC(N)/
     2  VISC(N))+RC(N)*VC(N)*UCN(N)/VISC(N))
      IF (S.GE.0.0001) GO TO 380
      A3=0.0
      B3=0.0
      C3=-C4/RC(N)
      D=D-REFAC*RNSH(N)*PPS*PFAC(N)/(RRS*VVM*VISC(N))
      GO TO 381
  380 CONTINUE
      A3=-REFAC*RNSH(N)*PPS/(RRS*UUS*VVM*VISC(N))
      B3=REFAC*RNSH(N)*PPS*XNSP(I)*XN(N)/(RRS*UUS*VVM*CNS*VISC(N))
      C3=-C4/RC(N)-REFAC*RNSH(N)*PSP/(RRS*VVM*UUS*VISC(N))
  381 CONTINUE
      DN1=DN(N)+DN(N-1)
      DN2=DN(N)-DN(N-1)
      A11(N)=(2.0-B1*DN(N))/DN(N-1)/DN1
      A13(N)=-(B3*DN(N))/DN(N-1)/DN1
      B11(N)=C1+(-2.0+B1*DN2)/DN(N)/DN(N-1)+A1/(DS*CRNI)
      B12(N)=C2
      B13(N)=            C3+B3*DN2/DN(N)/DN(N-1)+A3/(DS*CRNT)
      B14(N)=C4
      C11(N)=(2.0+B1*DN(N-1))/DN(N)   /DN1
      C13(N)=B3*DN(N-1)/DN(N)/DN1
      D1(N)=(A1*U1(N)+A3*P1(N))/(DS*CRNI)-D/CRNT-((1.0-CRNI)/CRNI)*
     1      (U1NN(N)+B1*U1N(N)+B3*P1N(N)+C1*U1(N)+C2*V1(N)+C3*P1(N)+
     2      C4*T1(N))
  400 CONTINUE
C  ENERGY COEFFICIENTS
      DO 420 N=2,IM
      A3=REFAC*RNSH(N)*UUS*PPS*VISCO*UC(N)/(VVM*RRS*TTS*CONO*VISC(N))
      A4=-REFAC*RNSH(N)*UUS*VISCO*RC(N)*UC(N)/(VVM*CONO*VISC(N))
      B1=2.0*UUS*UUS*VISCO*(UCN(N)-CK*RNSH(N)*UC(N))/(TTS*CONO)
      B3=REFAC*VISCO*PPS*(-UUS*RNSH(N)*XNSP(I)*XN(N)*UC(N)/(VVM*CNS)
     1   +VC(N))/(CONO*TTS*RRS*VISC(N))
      B4=REFAC*(UUS*XNSP(I)*RNSH(N)*VISCO*XN(N)*RC(N)*UC(N)/(VVM*CNS*
     1   CONO*VISC(N))-VISCO*RC(N)*VC(N)/(CONO*VISC(N)))+RVISC(N)+
     2   CK*RNSH(N)+RCSF(N)
      C1=RNSH(N)*VISCO*(REFAC*(RC(N)*(UUS*XNSP(I)*XN(N)*TCN(N)/(VVM*CNS)
     1   -UUS*TSP*TC(N)/(VVM*TTS)-UUS*TS(N)/VVM)/VISC(N)+UUS*UUS*PPS*
     2   PFAC(N)/(VVM*RRS*TTS*VISC(N)))-2.0*CK*UUS*UUS*(UCN(N)-CK*
     3   RNSH(N)*UC(N))/TTS)/CONO
```

```
      C2=REFAC*VTSCO*(-PC(N)*TCN(N)/TC(N)+PPS*PCN(N)/(RRS*TTS))/
     1   (CONO*VISC(N))
      C3=REFAC*VISCO*(RNSH(N)*UC(N)*((UUS*XNSP(I)*XN(N)*TCN(N)/(VVM*CNS)
     1   -UUS*TSP*TC(N)/(VVM*TTS)-UUS*TS(N)/VVM)/TC(N)+UUS*PSP/(VVM*RRS
     2   *TTS))-VC(N)*TCN(N)/TC(N))/(CONO*VISC(N))
      C4=-REFAC*VISCO*(RNSH(N)*RC(N)*UC(N)*(UUS*XNSP(I)*XN(N)*TCN(N)/
     1   (VVM*CNS)-UUS*TS(N)/VVM)-RC(N)          *VC(N)*TCN(N))/(CONO*TC(N)
     2   *VISC(N))
      D =-REFAC*VISCO*(RNSH(N)*(RC(N)*UC(N)*(UUS*XNSP(I)*XN(N)*TCN(N)/
     1   (VVM*CNS)-UUS*TSP*TC(N)/(VVM*TTS)-UUS*TS(N)/VVM)+UUS*PPS*UC(N)*
     2   UUS*PFAC(N)/(VVM*RRS*TTS))-RC(N)*VC(N)*TCN(N)+PPS*VC(N)*PCN(N)/
     3   (RRS*TTS))/(CONO*VISC(N))-UUS*UUS*VISCO*(UCN(N)-CK*RNSH(N)*
     4   UC(N))*(UCN(N)-CK*RNSH(N)*UC(N))/(TTS*CONO)
      DN1=DN(N)+DN(N-1)
      DN2=DN(N)-DN(N-1)
      A21(N)=-(B1*DN(N))/DN(N-1)/DN1
      A23(N)=-(B3*DN(N))/DN(N-1)/DN1
      A24(N)=(2.0-B4*DN(N))/DN(N-1)/DN1
      B21(N)=C1+B1*DN2/DN(N)/DN(N-1)
      B22(N)=C2
      B23(N)=C3             +B3*DN2/DN(N)/DN(N-1)+A3/(DS*CRNI)
      B24(N)=A4/(DS*CRNI)+B4*DN2/DN(N)/DN(N-1)-2.0/DN(N)/DN(N-1)+C4
      C21(N)=B1*DN(N-1)/DN(N)/DN1
      C23(N)=B3*DN(N-1)/DN(N)/DN1
      C24(N)=(2.0+B4*DN(N-1))/DN(N)/DN1
      D2(N)=(A3*P1(N)+A4*T1(N))/(DS*CRNI)-D/CRNI-((1.0-CRNI)/CRNI)*
     1      (T1NN(N)+B1*U1N(N)+B3*P1N(N)+B4*T1N(N)+C1*U1(N)+C2*V1(N)+
     2      C3*P1(N)+C4*T1(N))
  420 CONTINUE
C  CONTINUITY COEFFICIENTS
      DO 440 N=1,IM
      UCM=(UC(N)+UC(N+1))/2.0
      VCM=(VC(N)+VC(N+1))/2.0
      PCM=(PC(N)+PC(N+1))/2.0
      TCM=(TC(N)+TC(N+1))/2.0
      XNM=(XN(N)+XN(N+1))/2.0
      RNSHM=(RNSH(N)+RNSH(N+1))/2.0
      RCSFM=(RCSF(N)+RCSF(N+1))/2.0
      RCSFPM=(RCSFP(N)+RCSFP(N+1))/2.0
      USM=(US(N)+US(N+1))/2.0
      PSM=(PS(N)+PS(N+1))/2.0
      TSM=(TS(N)+TS(N+1))/2.0
      UCNM=(UC(N+1)-UC(N))/DN(N)
      VCNM=(VC(N+1)-VC(N))/DN(N)
      PCNM=(PC(N+1)-PC(N))/DN(N)
      TCNM=(TC(N+1)-TC(N))/DN(N)
      IF (S.GE.0.0001) GO TO 430
      A1=0.0
      A3=0.0
      A4=0.0
```

```
      B1=0.0
      B2=RRS*VVM*PCM*TCM
      B3=RRS*VVM*VCM*TCM
      B4=-RRS*VVM*VCM*PCM
      C1=4.0*RNSHM*RRS*UUS2*PCM*TCM/DS
      C2=RRS*VVM*(2.0*RNSHM*PCM*TCM+TCM*PCNM-PCM*TCNM)
      C3=RRS*(2.0*RNSHM*TCM*(2.0*UUS2*UCM/DS+VVM*VCM)+VVM*(TCM*VCNM-
     1   VCM*TCNM))
      C4=RRS*(2.0*RNSHM*PCM*(2.0*UUS2*UCM/DS+VVM*VCM)+VVM*(PCM*VCNM
     1   +VCM*PCNM))
      D=RRS*(-4.0*RNSHM*PCM*TCM*(2.0*UUS2*UCM/DS+VVM*VCM)+2.0*VVM*(VCM*
     1   PCM*TCNM-VCM*TCM*PCNM-PCM*TCM*VCNM))
      GO TO 431
  430 CONTINUE
      A1=RNSHM*RRS*UUS*TCM*PCM
      A3=RNSHM*RRS*UUS*UCM*TCM
      A4=-A3*PCM/TCM
      B1=-RNSHM*XNSP(I)*RRS*UUS*XNM*PCM*TCM/CNS
      B2=RRS*VVM*PCM*TCM
      B3=TCM*(RRS*VVM*VCM-RNSHM*RRS*UUS*XNSP(I)*XNM*UCM/CNS)
      B4=-B3*PCM/TCM
      C1=RRS*UUS*RNSHM*(-XNSP(I)*XNM*(TCM*PCNM-PCM*TCNM)/CNS+(TCM*PSM-
     1   PCM*TSM)+TCM*PCM*( USD/UUS+RSD/RRS+RCSFM*(RCSFPM/(CSF*CNS)-
     2   XNM*XNSP(I)/CNS)+(XNSPT-XNSP(I))/CNS))
      C2=RRS*VVM*(TCM*PCNM-PCM*TCNM+TCM*PCM*(CK*RNSHM+RCSFM))
      C3=RRS*VVM*(TCM*VCNM+(CK*RNSHM+RCSFM)*VCM*TCM-VCM*TCNM)+RRS*UUS*
     1   RNSHM*(TCM*(USM-XNM*XNSP(I)*UCNM/CNS)-UCM*TSM+TCM*UCM*(USD/UUS
     2   +RSD/RRS+RCSFM*(RCSFPM/(CSF*CNS)-XNM*XNSP(I)/CNS)+(XNSPT-
     3   XNSP(I))/CNS)+XNM*XNSP(I)*UCM*TCNM/CNS)
      C4=RRS*VVM*(PCM*VCNM+(CK*RNSHM+RCSFM)*VCM*PCM+VCM*PCNM)+RRS*UUS*
     1   RNSHM*(PCM*(USM-XNM*XNSP(I)*UCNM/CNS)+UCM*PSM+PCM*UCM*(USD/UUS
     2   +RSD/RRS+RCSFM*(RCSFPM/(CSF*CNS)-XNM*XNSP(I)/CNS)+(XNSPT-
     3   XNSP(I))/CNS)-XNM*XNSP(I)*UCM*PCNM/CNS)
      D=-2.0*RRS*VVM*(PCM*TCM*VCNM+(CK*RNSHM+RCSFM)*VCM*PCM*TCM+VCM*
     1   (TCM*PCNM-PCM*TCNM))     -2.0*RRS*UUS*RNSHM*(PCM*TCM*(USM-XNM*
     2   XNSP(I)*UCNM/CNS)+UCM*(TCM*PSM-PCM*TSM)+PCM*TCM*UCM*(USD/UUS+
     3    RSD/RRS+RCSFM*(RCSFPM/(CSF*CNS)-XNM*XNSP(I)/CNS)+(XNSPT-
     4   XNSP(I))/CNS)-XNM*XNSP(I)*UCM*(TCM*PCNM-PCM*TCNM)/CNS)
  431 CONTINUE
      A41(N+1)=A1+CRNI*(DS*C1-2.0*B1*DS/DN(N))
      A42(N+1)=    CRNI*(DS*C2-2.0*B2*DS/DN(N))
      A43(N+1)=A3+CRNI*(DS*C3-2.0*B3*DS/DN(N))
      A44(N+1)=A4+CRNI*(DS*C4-2.0*B4*DS/DN(N))
      B41(N+1)=A1+CRNI*(DS*C1+2.0*B1*DS/DN(N))
      B42(N+1)=    CRNI*(DS*C2+2.0*B2*DS/DN(N))
      B43(N+1)=A3+CRNI*(DS*C3+2.0*B3*DS/DN(N))
      B44(N+1)=A4+CRNI*(DS*C4+2.0*B4*DS/DN(N))
      D4(N+1)=-2.0*DS*D+(A1-(1.0-CRNI)*C1*DS)*(U1(N+1)+U1(N))-(1.0-CRNI)
     1      *C2*DS*(V1(N+1)+V1(N))+(A3-(1.0-CRNI)*C3*DS)*(P1(N+1)+P1(N))
     2       +(A4-(1.0-CRNI)*C4*DS)*(T1(N+1)+T1(N))-2.0*(1.0-CRNI)*
```

```
     3        (DS/DN(N))*(B1*(U1(N+1)-U1(N))+B2*(V1(N+1)-V1(N))+B3*
     4        (P1(N+1)-P1(N))+B4*(T1(N+1)-T1(N)))
  440 CONTINUE
C   N MOMENTUM COEFFICIENTS
      DO 460 N=2,IE
      PCM=(PC(N)+PC(N-1))/2.0
      UCM=(UC(N)+UC(N-1))/2.0
      VCM=(VC(N)+VC(N-1))/2.0
      TCM=(TC(N)+TC(N-1))/2.0
      XNM=(XN(N)+XN(N-1))/2.0
      RNSHM=(RNSH(N)+RNSH(N-1))/2.0
      PCNM=(PC(N)-PC(N-1))/DN(N-1)
      VCNM=(VC(N)-VC(N-1))/DN(N-1)
      VSM=(VS(N)+VS(N-1))/2.0
      VVV=VSM-XNSP(I)*XNM*VCNM/CNS+VSD*VCM/VVM
      C1=RNSHM*RRS*UUS*(-2.0*CK*UUS*PCM*UCM+THIN*VVM*VVV*PCM)
      A2=RNSHM*RRS*VVM*UUS*THIN*PCM*UCM
      B2=RRS*VVM*THIN*PCM*(-RNSHM*XNSP(I)*XNM*UUS*UCM/CNS+VVM*VCM)
      C2=RRS*VVM*THIN*PCM*(RNSHM*VSD*UUS*UCM/VVM+VVM*VCNM)
      B3=PPS*TCM
      C3=RRS*VVM*(RNSHM*(THIN*UUS*VVV*UCM-CK*UUS*UUS*UCM*UCM/VVM)+
     1    THIN*VVM*VCM*VCNM)
      C4=PPS*PCNM
      D=-PPS*TCM*PCNM+2.0*RRS*VVM*(RNSHM*UUS*PCM*UCM*(CK*UUS*UCM/VVM-
     1   THIN*VVV)-THIN*VVM*PCM*VCM*VCNM)
      B31(N-1)=CRNI*DS*C1
      B32(N-1)=A2+CRNI*(DS*C2-2.0*B2*DS/DN(N-1))
      B33(N-1)=CRNI*(DS*C3-2.0*B3*DS/DN(N-1))
      B34(N-1)=CRNI*DS*C4
      C31(N-1)=CRNI*DS*C1
      C32(N-1)=A2+CRNI*(DS*C2+2.0*B2*DS/DN(N-1))
      C33(N-1)=CRNI*(DS*C3+2.0*B3*DS/DN(N-1))
      C34(N-1)=CRNI*DS*C4
      D3(N-1)=-2.0*DS*D+(A2-(1.0-CRNI)*DS*C2)*(V1(N)+V1(N-1))-(1.0-CRNI)
     1    *DS*(C1*(U1(N)+U1(N-1))+C3*(P1(N)+P1(N-1))+C4*(T1(N)+T1(N-1)))
     2        -2.0*(1.0-CRNI)*DS*(B2*(V1(N)-V1(N-1))/DN(N-1)+B3*
     3          (P1(N)-P1(N-1))/DN(N-1))
  460 CONTINUE
      T2(1)=TW/TTS/CRNI-(1.0-CRNI)*T1(1)/CRNI
      V2IE=V2(IE)
      CALL DERSO ( TFACT,I)
      DO 802 N=1,IE
      R2(N)=P2(N)/T2(N)
  802 CONTINUE
      IF(NITT.GT.1) XNOO=XNSO
      XNSO=XNS
      IF(S.LE.0.0001) GO TO 717
      IF(NITT.EQ.1) XNS=1.01*XNS
      IF(NITT.EQ.1) GO TO 711
      XNS=XNS+(XNOO-XNS)*(VVS2-V2(IE))/(V2IE-V2(IE))
```

```
      GO TO 711
  717 CONTINUE
      AA=0.0
      BB=0.0
      DO 700 N=2,IE
      AA    =AA       +DN(N-1)*(R2(N-1)*U2(N-1)+R2(N)*U2(N))/2.
  700 BB    =BB       +DN(N-1)*(R2(N-1)*U2(N-1)*XN(N-1)
     1       +R2(N)*U2(N)*XN(N))/2.
      IF (S.GE.0.0001) GO TO 705
      AIA=8.*BB      *RRS2*UUS2*CSF2/DS-DS
      BIB=4.*AA      *RRS2*UUS2*RS2/DS-DS
      CIC=-DS
      ROT=BIB*BIB-AIA*CIC
      XNS = (-BIB + DSQRT(ROT)) / AIA
      CO2    =XNS*(RS2*AA    +XNS*CSF2*BB    )*RRS2*UUS2
      GO TO 711
  705 AIA=BB      *CSF2*RRS2*UUS2
      BIB=AA      *RS2*RRS2*UUS2/2.
      CIC=-CO1      +(RS+CNS*CSF)*((1.+CK*CNS)*RRS*VVS-XNSP(I)*RRS*UUS)*DS
      ROT=BIB*BIB-AIA*CIC
      XNS = (-BIB + DSQRT(ROT)) / AIA
      CO2    =XNS*(RS2*AA    +XNS*CSF2*BB    )*RRS2*UUS2
  711 IF (S.GE.0.0001) GO TO 715
      XNS1=XNS
  715 CNS=(XNS+XNS1)/2.0D0
      RSH1=RS2+XNS*CSF2
      RSH =RS +CNS*CSF
      XSH=XB-CNS*SIF
      YYYY(I)=RSH
      XXXX(I)=CNS
      IF(I.EQ.IEND) RMAXC=RSH
      IF(I.EQ.IEND) RMAX1=RSH1
      IF(I.EQ.IEND) RMAX=2.0*RMAX1-RMAXC
      IF( DABS(XNS-XNSO).GT.TFACT) TFACT= DABS(XNS-XNSO)
      IF (S.GE.0.0001) GO TO 641
      DO 640 N=1,IE
      RCSF(N)=CNS/(1.+CK*CNS*XN(N))
      U1(N)=U2(N)
      V1(N)=V2(N)
      P1(N)=P2(N)
      T1(N)=T2(N)
      R1(N)=R2(N)
      UC(N)=U2(N)
      VC(N)=V2(N)
      PC(N)=P2(N)
      TC(N)=T2(N)
      RC(N)=PC(N)/TC(N)
      U1N(N)=U2N(N)
      V1N(N)=V2N(N)
      P1N(N)=P2N(N)
```

```
      T1N(N)=T2N(N)
      UCN(N)=U2N(N)
      VCN(N)=V2N(N)
      PCN(N)=P2N(N)
      TCN(N)=T2N(N)
      U1NN(N)=U2NN(N)
      T1NN(N)=T2NN(N)
  640 CONTINUE
  641 CONTINUE
C
C  SOLVE N MOMENTUM EQUATION  ( STAGNATION REGION ONLY )
C**************************            **********************
C
      IF( S.GE. 0.0001) GO TO 800
      IF ( ITHIN.EQ.0) GO TO 716
      IF(NTIME.NE.1) GO TO 713
      VPG=VSP
      VVS2G=VVS2
      VVSG(1)=VVS
      DO 710 N=1,IE
      VG(N)=V2(N)
      VGS(N)=VS(N)
  710 VO(N)=VC(N)
      GO TO 714
  713 VVS2G=(VVSG(1)+VVSG(2))/2.0
      VPG  =(VSPP(1)+VSPP(2))/2.0
      DO 712 N=1,IE
      VG(N)=(VCI(N,1)+VCI(N,2))/2.0
      VGS(N)=(VCI(N,2)-VCI(N,1))/DS
  712 VO(N)=VCI(N,1)
  714 CONTINUE
      DO 720 N=2,IM
      VON(N)=(DN(N-1)*VO(N+1)/DN(N)-DN(N)*VO(N-1)/DN(N-1))/(DN(N)+
     1          DN(N-1))+(DN(N)-DN(N-1))*VO(N)/(DN(N)*DN(N-1))
      VGN(N)=(DN(N-1)*VG(N+1)/DN(N)-DN(N)*VG(N-1)/DN(N-1))/(DN(N)+
     1          DN(N-1))+(DN(N)-DN(N-1))*VG(N)/(DN(N)*DN(N-1))
  720 CONTINUE
      VON(IE)=VO(IE)*(DN(IM-1)+2.*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1         -VO(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2         +VO(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      VGN(IE)=VG(IE)*(DN(IM-1)+2.*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1         -VG(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2         +VG(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      VON(1)=-VO(1)*(DN(2)+2.*DN(1))/(DN(1)*(DN(1)+DN(2)))
     1         +VO(2)*(DN(2)+DN(1))/(DN(1)*DN(2))
     2         -VO(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      VGN(1)=-VG(1)*(DN(2)+2.*DN(1))/(DN(1)*(DN(1)+DN(2)))
     1         +VG(2)*(DN(2)+DN(1))/(DN(1)*DN(2))
     2         -VG(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
  716 CONTINUE
```

```
C
C*************************          *************************
C
      P21N(IE)=RRS2*UUS2**2 *CK2*XNS/(PPS2*(1.+CK2*XNS))
      P21(IE)=1.0
      PAN(IE)=P21N(IE)
      PA(IE)=1.
      PON(IE)=0.0
      PO(IE)=1.0
      CALL SHVALS(1.0D0,0.0D0,1.0D0,0.0D0,TTS0,VVS0,UUS0,PPS0,1)
      IF( ITHIN.EQ.0) GO TO 750
      P33N(IE )=-RRS2*VVS2G*VVS2G*((R2(IF )*VG(IE )-R2(IE )*U2(IE )*UUS2
     1          *XNSPM*XN(IE )/(VVS2G*(1.+CK2*XNS*XN(IE ))))*VON(IE )
     2          +UUS2*XNS*R2(IE )*U2(IE )*(VGS(IE )+VPG*VG(IE )/VVS2G)
     3          /(VVS2G*(1.+CK2*XNS*XN(IE ))))/PPS2
      P33(IE)=0.0
      PAN(IE)=PAN(IE)+P33N(IE)
      PON(IE )=VVS0(1)*PO(IE )*VO(IE )*VON(IE )/(PPS0*T2(IE ))
  750 CONTINUE
      KON=IM
      DO 810 N=1,IM
      P21N(KON)=RRS2*UUS2**2 *CK2*XNS*R2(KON)*U2(KON)**2 /(PPS2*(1.+CK2*
     1           XNS*XN(KON)))
      P21(KON)=P21(KON+1)-DN(KON)*(P21N(KON+1)+P21N(KON))/2.
      PAN(KON)=P21N(KON)
      PA(KON)=P21(KON)
      PON(KON)=0.0
      IF( ITHIN.EQ.0) GO TO 805
      P33N(KON)=-RRS2*VVS2G*VVS2G*((R2(KON)*VG(KON)-R2(KON)*U2(KON)*UUS2
     1          *XNSPM*XN(KON)/(VVS2G*(1.+CK2*XNS*XN(KON))))*VON(KON)
     2          +UUS2*XNS*R2(KON)*U2(KON)*(VGS(KON)+VPG*VG(KON)/VVS2G)
     3          /(VVS2G*(1.+CK2*XNS*XN(KON))))/PPS2
      P33(KON)=P33(KON+1)-DN(KON)*(P33N(KON+1)+P33N(KON))/2.
      PAN(KON)=PAN(KON)+P33N(KON)
      PA(KON)=PA(KON)+P33(KON)
      PON(KON)=VVS0(1)*P2(KON)*VO(KON)*VON(KON)/(PPS0*T2(KON))
  805 CONTINUE
      PO(KON)=PO(KON+1)-DN(KON)*(PON(KON+1)+PON(KON))/2.
  810 KON=KON-1
  800 CONTINUE
C
C*************************          *************************
C
      IF(NITT.GT.200) GO TO 3000
      IF(TFACT.GT.XFACT) GO TO 2000
      REFAC=RRS*VVM*CNS/(EPS*EPS*VISCO)
      CFCN=2.0*UUS*RRS*VVM*VISC(1)*(UCN(1)-CK*CNS*UC(1))/REFAC
      HEAT=TTS*RRS*VVM*(CONDKVISC(1)*TCN(1)/VISCO+UUS*UUS*VISC(1)*
     1      UC(1)*UCN(1)/TTS)/REFAC
      STAN=HEAT/(0.5+1.0/((GAM-1.0)*RMAC*RMAC)-TW)
```

```
      IF(I.LE.2) VVSG(I)=VVS
       IF(I.LE.2)  VSPP(I)=VSP
      CRNI=1.0D0
      DO 840 N=1,IE
      XM(N)=DSQRT((UUS*UUS*UC(N)*UC(N)+VVM*VVM*VC(N)*VC(N))/((GAM-1.0)
     1       *TTS*TC(N)))
      PO2PO1=1.0
      IF(XM(N).LE.1.0) GO TO 799
      PO2PO1=((GAM+1.0)*XM(N)*XM(N)/(2.0+(GAM-1.0)*XM(N)*XM(N)))**
     1       (GAM/(GAM-1.0))/(2.0*GAM*XM(N)*XM(N)/(GAM+1.0)-(GAM-1.0)/
     2       (GAM+1.0))**(1.0/(GAM-1.0))
  799 PITO(N)=PO2PO1*PC(N)*PPS*(1.0+(GAM-1.0)*XM(N)*XM(N)/2.0)**
     1        (GAM/(GAM-1.0))/POIP
      IF(I.EQ.1) VC(N)=VC(N)*VVS
      IF(I.EQ.1) V2(N)=VC(N)
      IF(I.EQ.1) V1(N)=V2(N)
      IF(I.LE.2) VCI(N,I)=VC(N)/VVS
      US(N)=(U2(N)-U1(N))/DS
      VS(N)=(V2(N)-V1(N))/DS
      PS(N)=(P2(N)-P1(N))/DS
      TS(N)=(T2(N)-T1(N))/DS
      UC(N)=(U2(N)+U1(N))/2.0
      VC(N)=(V2(N)+V1(N))/2.0
      PC(N)=(P2(N)+P1(N))/2.0
      TC(N)=(T2(N)+T1(N))/2.0
      UCN(N)=(U2N(N)+U1N(N))/2.0
      VCN(N)=(V2N(N)+V1N(N))/2.0
      PCN(N)=(P2N(N)+P1N(N))/2.0
      TCN(N)=(T2N(N)+T1N(N))/2.0
       RC(N)=PC(N)/TC(N)
      U1(N)=U2(N)
      V1(N)=V2(N)
      T1(N)=T2(N)
      R1(N)=R2(N)
      P1(N)=P2(N)
      T1N(N)=T2N(N)
      T1NN(N)=T2NN(N)
      U1N(N)=U2N(N)
      U1NN(N)=U2NN(N)
      V1N(N)=V2N(N)
      P1N(N)=P2N(N)
      CO1=CO2
  840 CONTINUE
      PWALL=PPS*PC(1)
      IF(S.LE.0.0001) GO TO 841
      CDP2=4.0*RS*SIF*PWALL
      CDF2=2.0*RS*CSF*CFCH
      CDPD=CDPD+(CDP1+CDP2)*DS/2.0
      CDFD=CDFD+(CDF1+CDF2)*DS/2.0
      CDP=CDPD/(RS*RS)
```

```
      CDF=CDFD/(RS*RS)
  841 IF(S.GE.0.0001) GO TO 842
      PWALO=PWALL
      CDF=0.0
      CDP=2.0*PWALL
  842 CDTOT=CDF+CDP
      CDP1=CDP2
      CDF1=CDF2
      PWRAT=PWALL/PWALO
C
C**************************              ************************
C
      XNS1=XNS
      CNS=XNS
      UUS1=UUS2
      VVS1=VVS2
      TTS1=TTS2
      PPS1=PPS2
      RRS1=RRS2
      RS1=RS2
      CSF1=CSF2
      CK1=CK2
      AD(I)=-2.0*CK*DTAN(ALPC-PHIC)-CNS*CKP/(1.0+CK*CNS)
      AB(I)=-2.0/DT
      AC(1)=-YNSPP(I)+(2.*CK*DTAN(ALPC-PHIC)+CNS*CKP/(1.0+CK*CNS))
     1       *YNSP(I)+2.0*(2.0*RSH-YNSH(I))/DT
      IF(NTIME.NE.NTIME1) GO TO 986
      IF((I-1)/IPRINT*IPRINT.NE.(I-1)) GO TO 986
      IF(I.GT.11) IPRINT=IPRIN1
      WRITE(6,908) S,XB,RS,CNS,XSH,RSH,EPS,NITT,NTIME
  908 FORMAT(//8X,'S',10X,'X',10X,'R',9X,'NSH',8X,'XSH',8X,'RSH',
     1       8X,'EPS',6X,'NO ITER',6X,'NTIME'/3X,7(F9.5,2X),I6,I12)
      WRITE(6,909) CFCH,HEAT,STAN,CDF,CDP,CDTOT,PWALL,PWRAT
  909 FORMAT(/6X,'CF',9X,'HEAT',7X,'STAN',7X,'CDF',8X,'CDP',8X,
     1       'CDTOT',5X,'PWALL',6X,'PW/PO'/3X,8(F9.5,2X))
      WRITE(6,910) UUS,VVS,PPS,TTS,RRS
  910 FORMAT(/6X,'UUS',8X,'VVS',8X,'PPS',8X,'TTS',8X,'RRS'/
     1       3X,5(F9.5,2X))
      WRITE(6,911)
      WRITE(6,912) (XN(N),UC(N),VC(N),PC(N),TC(N),RC(N),XM(N),
     1             PITO(N),N=1,IE)
  911 FORMAT(/8X,'N/NSH',7X,'U/USH',7X,'  V  ',7X,'P/PSH',7X,'T/TSH',
     1       7X,'R/RSH',7X,'MACH',8X,'PITO')
  912 FORMAT(4X,8F12.6)
  986 CONTINUE
      RS=RS2
      S=S+DS2
      CALL GEOM(S,DS2,RS,CK,CSF,SIF,XB)
      RS2=RS
      PHIC=DARCOS(CSF)
```

```
      RSH=RS+CNS*CSF
 5000 CONTINUE
C    ****$$$$$$$$$$****         N+1 TIME SWEEP   ***$$$$$$$$$$***
      CALL MANISH (DS,IEND,RMAX,CONV)
      IF(NTIME.NE.NTIME1) GO TO 987
      WRITE(6,914)
      WRITE(6,915) (YNSH(I),I=1,IEND1)
      WRITE(6,916) (XNSH(I),I=1,IEND1)
  915 FORMAT(/5X,'SHOCK RADUIS'/30(3X,10F10.4/))
  914 FORMAT(////////,3X,'SHOCK SHAPE'///4X,'FINAL SWEEP')
  916 FORMAT(/5X,'SHOCK THICKNESS',/30(3X,10F10.4/))
      WRITE(6,913) CONV,NTIME
  913 FORMAT( /3X,'FINAL SWEEP HAS CONVERGED TO : ',F12.4,2X,
     1     '(CHANGE IN RSH / RSH) ,',4X,'NO TIME CYCLES =',I8/)
      WRITE(6,917)
      WRITE(6,915) (YYYY(I),I=1,IEND1)
      WRITE(6,916) (XXXX(I),I=1,IEND1)
  917 FORMAT(//4X,'STAR SWEEP')
  987 CONTINUE
      IF(CONV.GT.0.0010000D0) GO TO 20
      IF(NTIME.EQ.NTIME1) GO TO 3000
      NTIME1=NTIME+1
       GO TO 20
 3000 CONTINUE
      STOP
      END
      SUBROUTINE SHVALS (SP, CP, SPB, CPB, TTSH, VRSH, URSH, PPSH, ID)
MOD 729.1,778.5
      IMPLICIT REAL*8 (A-H, O-Z)
      COMMON /INSH/ CONO  ,   GAM   ,   S     ,   UPSH  ,   XNS   ,
 ...
COMMAND? DEL 729.1,778.5
COMMAND? INSERT 728.1,777.1
  728.1  ? $
  777.1  ? $
COMMAND? LIST 704/779 UNNUMBERED MARKER=$
      RSH=RS+CNS*CSF                                   ...
COMMAND? LIST 704/778 UNNUMBERED MBRKER=$
```

```
      RSH=RS+CNS*CSF
 5000 CONTINUE
C    ****$$$$$$$$$$$*****        N+1  TIME SWEEP   ****$$$$$$$$$$****
      CALL MANISH (DS,IEND,RMAX,CONV)
      IF(NTIME.NE.NTIME1) GO TO 987
      WRITE(6,914)
      WRITE(6,915) (YNSH(I),I=1,IEND1)
      WRITE(6,916) (XNSH(I),I=1,IEND1)
  915 FORMAT(/5X,'SHOCK RADUIS'/30(3X,10F10.4/))
  914 FORMAT(/////////,3X,'SHOCK SHAPE'///4X,'FINAL SWEEP')
  916 FORMAT(/5X,'SHOCK THICKNESS',/30(3X,10F10.4/))
      WRITE(6,913) CONV,NTIME
  913 FORMAT( /3X,'FINAL SWEEP HAS CONVERGED TO : ',F12.4,2X,
     1     '(CHANGE IN RSH / RSH) ,',4X,'NO TIME CYCLES =',I8/)
      WRITE(6,917)
      WRITE(6,915) (YYYY(I),I=1,IEND1)
      WRITE(6,916) (XXXX(I),I=1,IEND1)
  917 FORMAT(//4X,'STAR SWEEP')
  987 CONTINUE
      IF(CONV.GT.0.00100000D0) GO TO 20
      IF(NTIME.EQ.NTIME1) GO TO 3000
      NTIME1=NTIME+1
       GO TO 20
 3000 CONTINUE
      STOP
      END
```

```
      SUBROUTINE SHVALS (SP, CP, SPB, CPB, TTSH, VRSH, URSH, PPSH, ID)
      IMPLICIT REAL*8 (A-H, O-Z)
      COMMON /INSH/ CONO  ,   GAM   ,   S     ,   UPSH  ,   XNS   ,
     1              EPS   ,   RMAC  ,   TPSH  ,   VISCO
      COMMON/OUTSH/ PPS   ,   RRS   ,   TTS   ,   UUS1  ,   VVS   ,
     1              PPS1  ,   RRS1  ,   TTS1  ,   UUS2  ,   VVS1  ,
     2              PSP   ,   RRS2  ,   TSP   ,   USP   ,   VVS2  ,
     3              PPS2  ,   RSP   ,   TTS2  ,   UUS   ,   VSP
      COMMON/MAINN/CRNI,DS,DN(111),XN(111),IM,IE
      GAMP   = GAM + 1.0D0
      GAMM   = GAM - 1.0D0
      RMACQ  = RMAC * RMAC
      EPSQ   = EPS  * EPS
      SPQ    = SP   * SP
      FOGQ   = 4.0D0/(GAMP*GAMP)
      DEN    = RMACQ * RMACQ * SPQ
      URSH = SP * CP / (SP + EPSQ * VISCO * UPSH/XNS)
      TTSH = ((URSH-CP)**2 + FOGQ*GAM*SPQ +(2.0D0/GAMM-FOGQ*GAMM)/RMACQ
     1                       - FOGQ/DEN)*0.5D0*SP/(SP+EPSQ*CONO*TPSH/XNS)
      PPSH =(2.0D0*SPQ - GAMM/(GAM*RMACQ)) / GAMP
      RRSH = GAM * PPSH / (GAMM * TTSH)
      VRSH = -SP / RRSH
      GO TO (20,5) , ID
    5 CONTINUE
      TTS2 = TTSH
      PPS2 = PPSH
      RRS2 = RRSH
      UUS2 =  URSH * SPB + VRSH * CPB
      VVS2 = -URSH * CPB + VRSH * SPB
      IF (S .GE. .0001) GO TO 10
      UUS1 = -UUS2
      VVS1 =  VVS2
      TTS1 =  TTS2
      PPS1 =  PPS2
      RRS1 =  RRS2
10    CONTINUE
      UUS=(UUS2+UUS1)/2.0D0
      VVS=(VVS2+VVS1)/2.0D0
      TTS=(TTS2+TTS1)/2.0D0
      PPS=(PPS2+PPS1)/2.0D0
      RRS=(RRS2+RRS1)/2.0D0
      USP = (UUS2 - UUS1) / DS
      VSP = (VVS2 - VVS1) / DS
      TSP = (TTS2 - TTS1) / DS
      PSP = (PPS2 - PPS1) / DS
      RSP = (RRS2 - RRS1) / DS
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GEOM(S,DS,RS,CK,CSF,SIF,XB)
C     FOR A HYPERBOLOID ASYMPTOTIC TO A CONE OF TOTAL INTERIOR ANGLE
C     OF 45 DEGREES
      IMPLICIT REAL*8 (A-H, O-Z)
      ANG=22.5D0/57.2957795D0
      C1=DTAN(ANG)**2
      C2=1.0D0+C1
      DERR = DS*DSQRT(C1*RS*RS+1.D0) /DSQRT(C2*RS*RS+1.D0)
      REXP=RS+DERR/2.0D0
 2001 CONTINUE
      DERR = DS*DSQRT(C1*REXP*REXP+1.D0) / DSQRT(C2*REXP*REXP+1.D0)
      DELT=RS+DERR/2.0D0-REXP
      IF(DELT.LE.0.00001) GO TO 2002
      REXP=RS+DERR/2.0D0
      GO TO 2001
 2002 RS=RS+DERR
      SQPT  = DSQRT(1.0D0 + C2 * RS * RS )
      XB=(-1.0D0+DSQRT(1.0D0+C1*RS*RS))/C1
      CK=1.0D0/(SQPT*SQPT*SQPT)
      CSF=RS/SQPT
      SIF   = DSQRT(1.0D0 - CSF * CSF)
      RETURN
      END
```

```
      SUBROUTINE DERSO ( TFACT,I )
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/DEQSS/ U2(111),V2(111),P2(111),T2(111),U2N(111),V2N(111),
     1            P2N(111),T2N(111),U2NN(111),T2NN(111)
      COMMON/DEQS/A11(111),A13(111),B11(111),B12(111),B13(111),B14(111),
     1   C11(111),C13(111), D1(111),A21(111),A23(111),A24(111),B21(111),
     2   B22(111),B23(111),B24(111),C21(111),C23(111),C24(111), D2(111),
     3   B31(111),B32(111),B33(111),B34(111),C31(111),C32(111),C33(111),
     4   C34(111), D3(111),A41(111),A42(111),A43(111),A44(111),B41(111),
     5   B42(111),B43(111),B44(111), D4(111)
      COMMON/MAINN/CRNI,DS,DN(111),XN(111),IM,IE
      COMMON/BOUNND/ DU(111),DV(111),DP(111),DT(111),
     1            EU(111),EV(111),EP(111),ET(111),
     2            FU(111),FV(111),FP(111),FT(111),
     3            GU(111),GV(111),GP(111),GT(111),
     4            HU(111),HV(111),HP(111),HT(111)
      N=IE
      N1=N-1
      DU(N)=0.0D0
      EU(N)=0.0D0
      FU(N)=0.0D0
      GU(N)=0.0D0
      HU(N)=U2(N)
      DV(N)=-A41(N)/B42(N)
      EV(N)=-A42(N)/B42(N)
      FV(N)=-A43(N)/B42(N)
      GV(N)=-A44(N)/B42(N)
      HV(N)=(D4(N)-B41(N)*U2(N)-B43(N)*P2(N)-B44(N)*T2(N))/B42(N)
      DP(N)=0.00
      EP(N)=0.00
      FP(N)=0.00
      GP(N)=0.00
      HP(N)=P2(N)
      DT(N)=0.0D0
      ET(N)=0.0D0
      FT(N)=0.0D0
      GT(N)=0.0D0
      HT(N)=T2(N)
      TFACT=0.0D0
      P20=P2(1)
      DO 30 J1=1,N1
      J=N-J1
      B31B=B31(J)+C31(J)*DU(J+1)+C32(J)*DV(J+1)+C33(J)*DP(J+1)+C34(J)*
     1 DT(J+1)
      B32B=B32(J)+C31(J)*EU(J+1)+C32(J)*EV(J+1)+C33(J)*EP(J+1)+C34(J)*
     1 ET(J+1)
      B33B=B33(J)+C31(J)*FU(J+1)+C32(J)*FV(J+1)+C33(J)*FP(J+1)+C34(J)*
     1 FT(J+1)
      B34B=B34(J)+C31(J)*GU(J+1)+C32(J)*GV(J+1)+C33(J)*GP(J+1)+C34(J)*
     1 GT(J+1)
```

```
 D3B =D3(J) -C31(J)*HU(J+1)-C32(J)*HV(J+1)-C33(J)*HP(J+1)-C34(J)*
1 HT(J+1)
 IF(J.EQ.1) GO TO 20
 B11B=B11(J)+C11(J)*DU(J+1)+C13(J)*DP(J+1)
 B12B=B12(J)+C11(J)*EU(J+1)+C13(J)*EP(J+1)
 B13B=B13(J)+C11(J)*FU(J+1)+C13(J)*FP(J+1)
 B14B=B14(J)+C11(J)*GU(J+1)+C13(J)*GP(J+1)
 D1B =D1(J) -C11(J)*HU(J+1)-C13(J)*HP(J+1)
 B21B=B21(J)+C21(J)*DU(J+1)+C23(J)*DP(J+1)+C24(J)*DT(J+1)
 B22B=B22(J)+C21(J)*EU(J+1)+C23(J)*EP(J+1)+C24(J)*ET(J+1)
 B23B=B23(J)+C21(J)*FU(J+1)+C23(J)*FP(J+1)+C24(J)*FT(J+1)
 B24B=B24(J)+C21(J)*GU(J+1)+C23(J)*GP(J+1)+C24(J)*GT(J+1)
 D2B =D2(J) -C21(J)*HU(J+1)-C23(J)*HP(J+1)-C24(J)*HT(J+1)
 XK1=B33B*B44(J)-B34B*B43(J)
 XK2=B32B*B44(J)-B34B*B42(J)
 XK3=B32B*B43(J)-B33B*B42(J)
 XK4=B23B*B44(J)-B24B*B43(J)
 XK5=B22B*B44(J)-B24B*B42(J)
 XK6=B22B*B43(J)-B23B*B42(J)
 XK7=B23B*B34B-B24B*B33B
 XK8=B22B*B34B-B24B*B32B
 XK9=B22B*B33B-B23B*B32B
 XK10=B31B*B44(J)-B34B*B41(J)
 XK11=B31B*B43(J)-B33B*B41(J)
 XK12=B21B*B44(J)-B24B*B41(J)
 XK13=B21B*B43(J)-B23B*B41(J)
 XK14=B21B*B34B-B24B*B31B
 XK15=B21B*B33B-B23B*B31B
 XK16=B31B*B42(J)-B32B*B41(J)
 XK17=B21B*B42(J)-B22B*B41(J)
 XK18=B21B*B32B-B22B*B31B
 B11C=B22B*XK1 B23B*XK2+B24B*XK3
 B21C=B12B*XK1-B13B*XK2+B14B*XK3
 B31C=B12B*XK4 B13B*XK5+B14B*XK6
 B41C=B12B*XK7-B13B*XK8+B14B*XK9
 DET=B11B*B11C-B21B*B21C+B31B*B31C-B41(J)*B41C
 D11C=B11C
 D21C=B12B*XK1-B13B*XK2+B14B*XK3
 D31C=B12B*XK4-B13B*XK5+B14B*XK6
 D41C=B12B*XK7 B13B*XK8+B14B*XK9
 D12C=B21B*XK1-B23B*XK10+B24B*XK11
 D22C=B11B*XK1-B13B*XK10+B14B*XK11
 D32C=B11B*XK4-B13B*XK12+B14B*XK13
 D42C=B11B*XK7-B13B*XK14+B14B*XK15
 D13C=B21B*XK2-B22B*XK10+B24B*XK16
 D23C=B11B*XK2-B12B*XK10+B14B*XK16
 D33C=B11B*XK5-B12B*XK12+B14B*XK17
 D43C=B11B*XK8-B12B*XK14+B14B*XK18
 D14C=B21B*XK3-B22B*XK11+B23B*XK16
 D24C=B11B*XK3-B12B*XK11+B13B*XK16
```

```
      D34C=B11B*XK6-B12B*XK13+B13B*XK17
      D44C=B11B*XK9-B12B*XK15+B13B*XK18
      DU(J)=(-A11(J)*D11C+A21(J)*D21C+A41(J)*D41C)/DET
      EU(J)=A42(J)*D41C/DET
      FU(J)=(A23(J)*D21C+A43(J)*D41C-A13(J)*D11C)/DET
      GU(J)=(A24(J)*D21C+A44(J)*D41C)/DET
      HU(J)=(D1B*D11C-D2B*D21C+D3B*D31C-D4(J)*D41C)/DET
      DV(J)=(A11(J)*D12C-A21(J)*D22C-A41(J)*D42C)/DET
      EV(J)=-A42(J)*D42C/DET
      FV(J)=-(A23(J)*D22C+A43(J)*D42C-A13(J)*D12C)/DET
      GV(J)=-(A24(J)*D22C+A44(J)*D42C)/DET
      HV(J)=(-D1B*D12C+D2B*D22C-D3B*D32C+D4(J)*D42C)/DET
      DP(J)=(-A11(J)*D13C+A21(J)*D23C+A41(J)*D43C)/DET
      EP(J)=A42(J)*D43C/DET
      FP(J)=(A23(J)*D23C+A43(J)*D43C-A13(J)*D13C)/DET
      GP(J)=(A24(J)*D23C+A44(J)*D43C)/DET
      HP(J)=(D1B*D13C-D2B*D23C+D3B*D33C-D4(J)*D43C)/DET
      DT(J)=(A11(J)*D14C-A21(J)*D24C-A41(J)*D44C)/DET
      ET(J)=-A42(J)*D44C/DET
      FT(J)=-(A23(J)*D24C+A43(J)*D44C-A13(J)*D14C)/DET
      GT(J)=-(A24(J)*D24C+A44(J)*D44C)/DET
      HT(J)=(-D1B*D14C+D2B*D24C-D3B*D34C+D4(J)*D44C)/DET
      GO TO 30
   20 P2(1)=(D3B-B31B* U2(1)- B32B*V2(1)-B34B*T2(1))/B33B
   30 CONTINUE
      TFACT= DABS( P2(1)-P20 )
      DO 40 J=2,N
      U20=U2(J)
      V20=V2(J)
      P20=P2(J)
      T20=T2(J)
      U2(J)=DU(J)*U2(J-1)+EU(J)*V2(J-1)+FU(J)*P2(J-1)+GU(J)*T2(J-1)+
     1     HU(J)
      V2(J)=DV(J)*U2(J-1)+EV(J)*V2(J-1)+FV(J)*P2(J-1)+GV(J)*T2(J-1)+
     1      HV(J)
      P2(J)=DP(J)*U2(J-1)+EP(J)*V2(J-1)+FP(J)*P2(J-1)+GP(J)*T2(J-1)+
     1     HP(J)
      T2(J)=DT(J)*U2(J-1)+ET(J)*V2(J-1)+FT(J)*P2(J-1)+GT(J)*T2(J-1)+
     1     HT(J)
      IF(DABS(U2(J)-U20).GT. TFACT ) TFACT=DABS(U2(J)-U20)
      IF(DABS(V2(J)-V20).GT. TFACT ) TFACT=DABS(V2(J)-V20)
      IF(DABS(P2(J)-P20).GT. TFACT ) TFACT=DABS(P2(J)-P20)
      IF(DABS(T2(J)-T20).GT. TFACT ) TFACT=DABS(T2(J)-T20)
   40 CONTINUE
      DO 50 N=2,IM
      U2NN(N)=2.0D0*(U2(N+1)/DN(N)+U2(N-1)/DN(N-1))/(DN(N)+DN(N-1))
     1        -2.0D0*U2(N)/(DN(N)*DN(N-1))
      T2NN(N)=2.0D0*(T2(N+1)/DN(N)+T2(N-1)/DN(N-1))/(DN(N)+DN(N-1))
     1        -2.0D0*T2(N)/(DN(N)*DN(N-1))
      U2N(N)=(DN(N-1)*U2(N+1)/DN(N)-DN(N)*U2(N-1)/DN(N-1))/(DN(N)+
```

```
     1          DN(N-1))+(DN(N)-DN(N-1))*U2(N)/(DN(N)*DN(N-1))
      V2N(N)=(DN(N-1)*V2(N+1)/DN(N)-DN(N)*V2(N-1)/DN(N-1))/(DN(N)+
     1          DN(N-1))+(DN(N)-DN(N-1))*V2(N)/(DN(N)*DN(N-1))
      P2N(N)=(DN(N-1)*P2(N+1)/DN(N)-DN(N)*P2(N-1)/DN(N-1))/(DN(N)+
     1          DN(N-1))+(DN(N)-DN(N-1))*P2(N)/(DN(N)*DN(N-1))
      T2N(N)=(DN(N-1)*T2(N+1)/DN(N)-DN(N)*T2(N-1)/DN(N-1))/(DN(N)+
     1          DN(N-1))+(DN(N)-DN(N-1))*T2(N)/(DN(N)*DN(N-1))
   50 CONTINUE
      U2N(1)=-U2(1)*(DN(2)+2.ODO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1          +U2(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2          -U2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      V2N(1)=-V2(1)*(DN(2)+2.ODO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1          +V2(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2          -V2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      P2N(1)=-P2(1)*(DN(2)+2.ODO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1          +P2(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2          -P2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      T2N(1)=-T2(1)*(DN(2)+2.ODO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1          +T2(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2          -T2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      U2N(IE)=U2(IE)*(DN(IM-1)+2.ODO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1           -U2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2           +U2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      V2N(IE)=V2(IE)*(DN(IM-1)+2.ODO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1           -V2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2           +V2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      P2N(IE)=P2(IE)*(DN(IM-1)+2.ODO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1           -P2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2           +P2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      T2N(IE)=T2(IE)*(DN(IM-1)+2.ODO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1           -T2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2           +T2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      U2NN(1)=-U2N(1)*(DN(2)+2.DO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1           +U2N(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2           -U2N(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      T2NN(1)=-T2N(1)*(DN(2)+2.DO*DN(1))/(DN(1)*(DN(2)+DN(1)))
     1           +T2N(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
     2           -T2N(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
      U2NN(IE)=U2N(IE)*(DN(IM-1)+2.DO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1            -U2N(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2            +U2N(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      T2NN(IE)=T2N(IE)*(DN(IM-1)+2.DO*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
     1            -T2N(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
     2            +T2N(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
      RETURN
      END
```

```
      SUBROUTINE DERIV(DS,IEND,ID)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/SHCKG/XNSH(202),XNSP(202),XNSPP(202),YNSH(202),YNSP(202),
     1             YNSPP(202),A1(202),A2(202),A3(202)
      IEND1=IEND+1
      XS=0.0
      RS=0.0D0
      DO 10 I=2,IEND1
      CALL GEOM(XS,DS,RS,CK,CSF,SIF,XB)
      IF(ID.EQ.1) YNSH(I)=RS+XNSH(I)*CSF
      IF(ID.EQ.2) XNSH(I)=(YNSH(I)-RS)/CSF
      XS=XS+DS
   10 CONTINUE
      IF(ID.EQ.2) XNSH(1)=(4.0*XNSH(2)-XNSH(3))/3.0
      XNSP(1)=0.0
      YNSPP(1)=(2.0*YNSH(1)-5.0*YNSH(2)+4.0*YNSH(3)-YNSH(4))/(DS*DS)
      XNSPP(1)=(2.0*XNSH(1)-5.0*XNSH(2)+4.0*XNSH(3)-XNSH(4))/(DS*DS)
      YNSP(1)=(4.0*YNSH(2)-YNSH(3)-3.0*YNSH(1))/(2.0*DS)
      DO 20 I=2,IEND
      XNSP(I)=(XNSH(I+1)-XNSH(I-1))/(2.0*DS)
      YNSP(I)=(YNSH(I+1)-YNSH(I-1))/(2.0*DS)
      XNSPP(I)=(XNSH(I+1)-2.0*XNSH(I)+XNSH(I-1))/(DS*DS)
      YNSPP(I)=(YNSH(I+1)-2.0*YNSH(I)+YNSH(I-1))/(DS*DS)
   20 CONTINUE
      XNSP(IEND1)=(3.0*XNSH(IEND1)-4.0*XNSH(IEND)+XNSH(IEND-1))/(2.0*DS)
      YNSP(IEND1)=(3.0*YNSH(IEND1)-4.0*YNSH(IEND)+YNSH(IEND-1))/(2.0*DS)
      XNSPP(IEND1)=(2.0*XNSH(IEND1)-5.0*XNSH(IEND)+4.0*XNSH(IEND1-2)-
     1             XNSH(IEND1-3))/(DS*DS)
      YNSPP(IEND1)=(2.0*YNSH(IEND1)-5.0*YNSH(IEND)+4.0*YNSH(IEND1-2)-
     1           YNSH(IEND1-3))/(DS*DS)
      RETURN
      END
```

```
      SUBROUTINE MANISH (DS,IEND,RMAX,CONV)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/SHCK6/XNSH(202),XNSP(202),XNSPP(202),YNSH(202),YNSP(202),
     1             YNSPP(202),A1(202),A2(202),A3(202)
      DIMENSION E(202),F(202)
      IM=IEND
      E(1)=0.0
      F(1)=0.0
      DO 10 I=2,IM
      A=1.0/(DS*DS)-A1(I)/(2.0*DS)
      B=-2.0/(DS*DS)+A2(I)
      C=1.0/(DS*DS)+A1(I)/(2.0*DS)
      D=-A3(I)
      E(I)=-C/(B+A*E(I-1))
      F(I)=(D-A*F(I-1))/(B+A*E(I-1))
   10 CONTINUE
      CONV=0.0
      KON=IM
      YNSH(IEND+1)=RMAX
      DO 20 I=1,IM
      YO=YNSH(KON)
      YNSH(KON)=E(KON)*YNSH(KON+1)+F(KON)
      CONV1=DABS(YNSH(KON)-YO)
      IF(I.NE.IM) CONV1=CONV1/YO
      IF(CONV1.GT.CONV) CONV=CONV1
   20 KON=KON-1
      CALL DERIV(DS,IEND,2)
      RETURN
      END
```

```
      BLOCK DATA
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/SHCKD/XNSH(202),XNSP(202),XNSPP(202),YNSH(202),YNSP(202),
     1             YNSPP(202),AD(202),AB(202),AC(202)
      DATA XNSH/202*0.1072D0/
      END
```

```
        INPUT DATA


  21.750       0.050    430.000    351.800
 1   51   21     1     1
0.100 10.000  1.400   0.200 0.100D-05
```

Program Output:

In the following pages, parts of the output are presented.

SHOCK LAYER PROGRAM

INPUT DATA

MINF	TINF	REYIN	TW/TO	GAM	PR
21.75	351.80	430.00	0.05	1.4	0.70

FULL-SHOCK LAYER   NO. OF STEPS IN N = 51   NO. OF STEPS IN S = 21   S STEP SIZE =0.100   T STEP SIZE = 10.0

INITIAL SHOCK SHAPE

SHOCK THICKNESS

0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072
0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072	0.1072
0.1072	0.1072								

SHOCK RADIUS

0.0	0.1105	0.2196	0.3261	0.4291	0.5282	0.6234	0.7146	0.8021	0.8862
0.9672	1.0453	1.1209	1.1941	1.2652	1.3344	1.4019	1.4678	1.5322	1.5953
1.6572	1.7179								

SHOCK SHAPE

FINAL SWEEP

SHOCK RADIUS

0.0	0.1125	0.2239	0.3332	0.4397	0.5432	0.6436	0.7410	0.8356	0.9276
1.0172	1.1046	1.1901	1.2738	1.3558	1.4363	1.5153	1.5929	1.6693	1.7444
1.8182	1.8908								

SHOCK THICKNESS

0.1269	0.1275	0.1293	0.1324	0.1367	0.1422	0.1488	0.1565	0.1651	0.1746
0.1848	0.1957	0.2072	0.2191	0.2314	0.2441	0.2570	0.2700	0.2832	0.2964
0.3094	0.3222								

FINAL SWEEP HAS CONVERGED TO :   0.0009 (CHANGE IN RSH / RSH) ,   NO TIME CYCLES =   18

STAR SWEEP

SHOCK RADIUS

0.0	0.1107	0.2203	0.3285	0.4346	0.5382	0.6392	0.7375	0.8331	0.9260
1.0164	1.1045	1.1904	1.2743	1.3562	1.4365	1.5152	1.5924	1.6684	1.7433
1.8173	0.0								

SHOCK THICKNESS

0.1088	0.1090	0.1112	0.1162	0.1227	0.1308	0.1400	0.1501	0.1608	0.1720
0.1836	0.1955	0.2075	0.2197	0.2320	0.2443	0.2567	0.2692	0.2819	0.2948
0.3082	0.0								

S	X	R	RSH	XSH	RSH	EPS	NO ITER	NTIME
2.00000	1.12654	1.57168	0.31496	0.54020	1.81733	0.22339	6	18

CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.14601	0.05405	0.11269	0.15744	1.03505	1.19248	0.35755	0.38234

UUS	VVS	PPS	TTS	RRS
0.72558	-0.03027	0.40534	0.24158	5.87244

N/NSH	U/USH	V	P/PSH	T/TSH	R/RSH	MACH	PITO
0.0	0.0	0.0	0.882110	0.103030	8.561707	0.0	0.392036
0.020000	0.080374	0.000011	0.882162	0.228443	3.861623	0.395830	0.436770
0.040000	0.141340	-0.000037	0.882400	0.317955	2.775239	0.589749	0.496236
0.060000	0.193609	-0.000150	0.882817	0.391015	2.257756	0.728196	0.558329
0.080000	0.240453	-0.000323	0.883399	0.453067	1.947240	0.839330	0.622756
0.100000	0.283393	-0.000551	0.884136	0.508763	1.737747	0.933807	0.689720
0.120000	0.323283	-0.000830	0.885020	0.558018	1.586006	1.016871	0.759409
0.140000	0.360663	-0.001157	0.886042	0.602442	1.470750	1.091513	0.831279
0.160000	0.395896	-0.001527	0.887195	0.642801	1.380202	1.159609	0.903932
0.180000	0.429242	-0.001937	0.888474	0.679644	1.307265	1.222415	0.976610
0.200000	0.460894	-0.002384	0.889872	0.713391	1.247384	1.280819	1.048937
0.220000	0.491001	-0.002865	0.891385	0.744377	1.197491	1.335470	1.120709
0.240000	0.519682	-0.003377	0.893006	0.772873	1.155437	1.386860	1.191807
0.260000	0.547029	-0.003919	0.894731	0.799105	1.119666	1.435371	1.262153
0.280000	0.573123	-0.004487	0.896555	0.823264	1.089025	1.481306	1.331695
0.300000	0.598030	-0.005079	0.898473	0.845512	1.062637	1.524910	1.400391
0.320000	0.621808	-0.005695	0.900481	0.865994	1.039823	1.566387	1.468212
0.340000	0.644506	-0.006330	0.902573	0.884834	1.020048	1.605907	1.535129
0.360000	0.666170	-0.006985	0.904747	0.902144	1.002885	1.643615	1.601123
0.380000	0.686840	-0.007656	0.906997	0.918023	0.987990	1.679637	1.666174
0.400000	0.706555	-0.008344	0.909320	0.932560	0.975079	1.714082	1.730269
0.420000	0.725349	-0.009045	0.911711	0.945838	0.963919	1.747051	1.793398
0.440000	0.743257	-0.009759	0.914167	0.957929	0.954316	1.778630	1.855554
0.460000	0.760309	-0.010484	0.916683	0.968903	0.946105	1.808902	1.916735
0.480000	0.776537	-0.011218	0.919258	0.978820	0.939149	1.837940	1.976944
0.500000	0.791972	-0.011961	0.921886	0.987740	0.933329	1.865815	2.036188
0.520000	0.806642	-0.012711	0.924566	0.995715	0.928545	1.892591	2.094478
0.540000	0.820576	-0.013467	0.927294	1.002795	0.924709	1.918330	2.151831
0.560000	0.833803	-0.014228	0.930067	1.009027	0.921746	1.943092	2.208268
0.580000	0.846352	-0.014992	0.932883	1.014454	0.919591	1.966932	2.263816
0.600000	0.858251	-0.015758	0.935740	1.019118	0.918187	1.989905	2.318503
0.620000	0.869527	-0.016525	0.938636	1.023055	0.917483	2.012064	2.372366
0.640000	0.880207	-0.017293	0.941565	1.026304	0.917436	2.033458	2.425444
0.660000	0.890320	-0.018059	0.944537	1.028899	0.918007	2.054136	2.477779
0.680000	0.899892	-0.018823	0.947540	1.030873	0.919163	2.074146	2.529417
0.700000	0.908950	-0.019584	0.950575	1.032256	0.920872	2.093534	2.580409
0.720000	0.917521	-0.020342	0.953644	1.033080	0.923108	2.112344	2.630808
0.740000	0.925630	-0.021094	0.956744	1.033372	0.925846	2.130619	2.680668
0.760000	0.933303	-0.021842	0.959875	1.033161	0.929067	2.148399	2.730047
0.780000	0.940566	-0.022583	0.963038	1.032472	0.932750	2.165727	2.779005
0.800000	0.947442	-0.023317	0.966233	1.031332	0.936878	2.182639	2.827601
0.820000	0.953957	-0.024045	0.969458	1.029764	0.941437	2.199173	2.875898
0.840000	0.960133	-0.024765	0.972716	1.027793	0.946412	2.215364	2.923959
0.860000	0.965994	-0.025478	0.976006	1.025441	0.951792	2.231248	2.971847
0.880000	0.971563	-0.026183	0.979329	1.022729	0.957565	2.246856	3.019625
0.900000	0.976860	-0.026881	0.982686	1.019678	0.963722	2.262221	3.067357
0.920000	0.981908	-0.027571	0.986077	1.016308	0.970254	2.277371	3.115106
0.940000	0.986726	-0.028254	0.989503	1.012638	0.977154	2.292337	3.162937
0.960000	0.991335	-0.028931	0.992965	1.008686	0.984415	2.307145	3.210911
0.980000	0.995753	-0.029602	0.996464	1.004468	0.992032	2.321823	3.259093
1.000000	1.000000	-0.030268	1.000000	1.000000	1.000000	2.336396	3.307544

SYMBOLS

c^*	viscosity law constant, $c^* = 198.6°R$
C_f	skin friction coefficient, $2\tau^*/(\rho_\infty^* u_\infty^{*2})$
C_p^*	specific heat of constant pressure
H	nondimensional total enthalpy, H^*/u_∞^{*2}
k	thermal conductivity
M_∞	free stream Mach number
n	coordinate measured normal to the body, nondimensionalized by the body nose radius
n_{sh}	shock stand off distance normal to the body surface
p	nondimensional pressure, $p^*/(\rho_\infty^* u_\infty^{2*})$
q	nondimensional heat transfer, $q^*/(\rho_\infty^* u_\infty^{*3})$
r	nondimensional axisymmetric radius
R	defined as $y_B + n_{sh}\cos\phi$
s	nondimensional surface distance coordinate
St	Stanton number, $St = q_w/(H_o-H_w)$
t	time, and also used for normalized temperature, T/T_{sh}
T	nondimensional temperature, $T = T^*/(u_\infty^{*2}/C_p^*)$
T_∞^*	free stream temperature
u	nondimensional velocity component tangent to the body surface, u^*/u_∞^*.
u_∞^*	free stream velocity
$\tilde{u}$	nondimensional component of velocity aft and tangent to the shock interface
v	nondimensional velocity component normal to the body surface, v^*/u_∞^*
$\tilde{v}$	nondimensional component of velocity aft and normal to shock interface
x_B	axial distance for body surface measured from stagnation point

x_{sh} defined as $x_B - n_s \sin\phi$

y_B normal distance for body surface measured from axis

α shock angle, see Figure 1

β angle defined in Figure 1

γ ratio of specific heats

ε perturbation parameter, $\varepsilon = [\mu^*(u_\infty^{*2}/C_p^*)/\rho_\infty^* u_\infty^* a^*]^{1/2}$

κ nondimensional surface curvature

μ nondimensional coefficient of viscosity, $\mu = \mu^*/\mu^*(u_\infty^{*2}/C_p^*)$

ρ nondimensional density, $\rho = \rho^*/\rho_\infty^*$

ρ_∞ free stream density

τ nondimensional shear stress, $\tau^*/(\rho_\infty^* u_\infty^{*2})$

ϕ body angle defined in Figure 1

σ Prandtl number, $\sigma = \mu C_p/K$

ξ nondimensional surface distance coordinate, s

η normalized normal coordinate, n/n_{sh}

Subscripts

1 wall value

0 stagnation conditions

n used for normal derivatives

N shock value

s used for longitudinal derivatives

sh conditions immediately behind the shock wave

∞ free stream conditions

w wall value

ξ used for longitudinal derivatives

η used for normal derivatives

Superscripts

'	longitudinal derivative, d/ds
-	physical quantities normalized by their shock values
*	dimensional quantities, also used for first sweep of ADI numerical scheme
j	0 for plane flow and 1 for axisymmetric flow